Fund. of Digital Communications
Ch. 3: Digital Modulation

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Outline

- 3-1 Pulse Amplitude Modulation
  - Baseband and Bandpass Signals
  - One-, Two-, and Multidimensional Signals
  - QAM and Complex Equivalent Baseband Signals

- 3-2 Pulse Shaping and ISI-free Transmission
  - Signal Spectrum
  - Nyquist Pulse Shaping
References and Figures

- Figures refer to Chapter 7 of

- References to figures denoted as [7.x]

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3-1 Pulse Amplitude Modulation (PAM)

- In practice: TX signal is a stream of symbols
  \[ s(t) = \sum_{i=-\infty}^{\infty} s[i](t - iT) \]

  where \( s[i](t) \in \{ s_m(t) \}_{m=1}^{M} \) represents symbol \( s[i] \), taken from an \( M \)-ary alphabet \( \{ s_m(t) \} \)

- Basic assumption:
  - Consecutive symbols do not interfere
  - Thus we can concentrate on one single symbol
  - Symbol index \( i \) is dropped “without loss of generality”; the TX signal is \( s(t) \in \{ s_m(t) \}_{m=1}^{M} \)
Pulse Amplitude Modulation (PAM)

- Transmission of information through modulation of signal amplitude
  - Signal shape $g_T(t)$ is tailored to channel

- Baseband signals – for baseband channels [7.4]
  - Binary antipodal modulation; selects amplitude of a pulse waveform $g_T(t)$
    
    "1" $\Rightarrow A$ for $s_1(t) = A g_T(t)$
    
    "0" $\Rightarrow -A$ for $s_2(t) = -A g_T(t)$

- Bit rate $R_b$, bit interval $T_b$ (= symbol interval)
  
  $$R_b = \frac{1}{T_b}$$

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PAM, Baseband (cont’d)

![Binary PAM signals](Figure 7.4)
PAM, Baseband (cont’d)

- \( M \)-ary PAM [7.5]
  - Usually: \( M = 2^k \) for integer \( k \); \( k \) ... nb. bits/symbol
  - Symbol interval: \( T = k/R_b = kT_b \) [7.6]

- (Set of) \( M \) signal waveforms [7.7]
  
  \[ s_m(t) = A_m g_T(t), \text{ for } m \in \{1, 2, ..., M\}, 0 \leq t \leq T \]

  - Pulse shape \( g_T(t) \) determines signal spectrum [7.9]
  - Energy (can vary for \( m \in \{1, 2, ..., M\} \))
    
    \[ E_m = \int_0^T s_m^2(t) dt = A_m^2 \int_0^T g_T^2(t) dt = A_m^2 E_g \]

    \( E_g \) ... energy of pulse \( g_T(t) \)

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**Figure 7.5** \( M = 4 \) PAM signal waveforms.

**Figure 7.6** Relationship between the symbol interval and the bit interval.

\( T_b = \text{bit interval} \)
\( T = \text{symbol interval} \)
PAM, Baseband (cont’d)

![Graph of signal pulse for PAM](image)

**PAM, Bandpass (Passband)**

- **Bandpass signals** – for bandpass channels
- **Carrier modulation** [7.8]
  - Multiplication of $s_m(t)$ by carrier $\cos(2\pi f_c t)$
    \[ u_m(t) = s_m(t) \cos(2\pi f_c t) = A_m g_T(t) \cos(2\pi f_c t), \]
    for $m \in \{1, 2, \ldots, M\}$,
    $f_c$ ... carrier frequency (center frequency)
  - in frequency domain [7.9]:
    \[ U_m(f) = \frac{A_m}{2} [G_T(f - f_c) + G_T(f + f_c)] \]
    - DSB-SC-AM (Dual sideband, suppressed carr. AM)
    - Channel bandwidth $2W$ (doubled w.r.t. baseband!)
PAM, Baseband (cont’d)

Figure 7.8 Amplitude modulates sinusoidal carrier by the baseband signal.

Figure 7.9 Spectra of (a) baseband and (b) amplitude-modulated signals.
Definitions of Bandwidth

Definitions

- Absolute bandwidth
- 3-dB bandwidth
- Equivalent bandwidth (BW of block spectrum with equal energy and const. amplitude as at $f_c$)
- First spectral zero (BW of main lobe)

Time-bandwidth product is constant!

- E.g. first zero of rectangular pulse:
- Interval $T$ vs. first zero $B_z$ of its Fourier transform:

$$B_z = \frac{1}{T}, \text{ hence } TB_z = 1$$

Geometric Representation in Signal Space

- PAM Signals are one-dimensional

$$s_m(t) = s_m \psi(t)$$

- Baseband:

$$\psi(t) = \frac{1}{\sqrt{E_g}} g_T(t), \quad 0 \leq t \leq T$$

$$s_m = \sqrt{E_g A_m}, \quad m \in \{1, 2, ..., M\}$$

- Bandpass:

$$\psi(t) = \sqrt{\frac{2}{E_g}} g_T(t) \cos 2\pi f_c t$$

$$s_m = \sqrt{E_g / 2} A_m, \quad m \in \{1, 2, ..., M\}$$
Geometric Representation (cont’d)

- **Euclidean distance**
  \[ d_{mn} = \sqrt{|s_m - s_n|^2} \]

- **Energy of PAM signals (baseband)**
  \[ E_m = s_m^2 = E_g A_m^2, \quad m \in \{1, 2, \ldots M\} \]

- **e.g.: symmetric PAM [7.11]**

Two- (and Multidimensional) Signals

- **Simultaneous PAM** of two (or more) basis functions
  - yields additional points in \( N \)-dim. signal space; each representing a signal waveform

- **orthogonal** signals
  - \( M \)-ary symbols are represented by \( N = M \) orthogonal waveforms
  - (see [7.12]–[7.14] for \( M = 2 \))
Two-dimensional Signals

Figure 7.12 Two sets of orthogonal signals.

Figure 7.13 The two signal vectors corresponding to the signals waveforms $s_1(t)$ and $s_2(t)$.

Figure 7.14 The two signal vectors corresponding to the signal waveforms $s'_1(t)$ and $s'_2(t)$. 
Two-dimensional Signals

- **bi-orthogonal** signals [7.15]
  - binary antipodal PAM of the basis functions
  - $M = 4$-ary signals with equal energies
  - add signal vectors with inverted polarities:

![Figure 7.15](image1)

Two-dimensional Signals

- $M = 8$-ary signals with equal energies

![Figure 7.16](image2)

Two-dimensional Signals

- $M = 8$-ary signals with (two) different energies

![Figure 7.17](image3)
Two-dimensional Bandpass Signals–QAM

- important example for 2D-signals: (digital) QAM
  - PAM of the orthogonal carriers
    \[ u_m(t) = A_{mc} g_T(t) \cos 2\pi f_c t - A_{ms} g_T(t) \sin 2\pi f_c t \]
  - in geometric representation:
    \[ u_m(t) = s_{m1} \psi_1(t) + s_{m2} \psi_2(t), \text{ with} \]
    \[ \psi_1(t) = \sqrt{2/E_g} g_T(t) \cos 2\pi f_c t \]
    \[ \psi_2(t) = -\sqrt{2/E_g} g_T(t) \sin 2\pi f_c t \]
    \[ s_m = [s_{m1}, s_{m2}]^T = [\sqrt{E_s} A_{mc}, \sqrt{E_s} A_{ms}]^T \]

- QAM signals have a complex-valued equivalent baseband representation; no BW loss! (see Chapter 2)
Two-dimensional Bandpass Signals–QAM (cont’d)

- Functional block diagram of a (digital) QAM modulator

![Functional block diagram of a (digital) QAM modulator](image)

Figure 7.22 Functional block diagram of modulator for QAM.

Figure 7.23 (a) Rectangular signal-space constellations for QAM. (b, c) Examples of combined PAM-PSK signal-space constellations.
Multidimensional Signals

■ Two-dimensional case: $M = 2^k$ signals have been constructed in 2D

■ Multidimensional case: (OPAM)
  ♦ construct $N$ orthogonal signals
  ♦ define signal points in these dimensions

$$s_\mu(t) = \sum_{n=1}^{N} a_{m,n} g_n(t)$$

$\{g_n(t)\}_{n=1}^{N}$ ... $N$ orthogonal waveforms (basis)
$\{a_{m,n} \in \mathbb{R} (\text{or } \mathbb{C})\}$ ... PAM (or QAM) symbols ($M$-ary) for $n$-th waveform; $m \in 1, 2, \ldots, M$
$\{s_\mu(t)\}_{\mu=1}^{MN}$ ... $MN$-ary set of waveforms

Multidimensional Signals

■ OFDM: parallel transmission to enlarge symbol duration against inter-symbol-interference (ISI)

$$g_n(t) = \frac{1}{\sqrt{T}} e^{j2\pi nt/T} w(t) \ldots \text{n-th subcarrier at } f = n/T$$

$$s_\mu(t) = \frac{1}{\sqrt{T}} \sum_{n=-N/2}^{N/2-1} a_{m,n} e^{j2\pi nt/T} w(t)$$

$\{g_n(t)\}$ ... orthogonal subcarriers (Fourier basis)
$w(t)$ ... window function (e.g. rectangular)
$\{a_{m,n} \in \mathbb{C}\}$ ... QAM symbols (e.g. QPSK, 16/64-QAM)
♦ the symbols $s_\mu(t)$ are $MN$-ary ($N$ subcarriers)
Multidimensional Signals

- **Spread Spectrum/CDMA**: few points in $N \gg 1$ dimensional space (large TB-product)
  - divide $T$ into $N$ chip intervals $T_c = T/N$
  - modulate chip waveform with “spreading code” $\{c_n\}$

\[
g_T(t) = \sum_{n=0}^{N-1} c_n g_c(t - nT_c); \quad s_m(t) = a_m g_T(t) \text{ (PAM/QAM)}
\]

- chip waveform $g_c(t)$ has $N$-fold bandwidth $\propto 1/T_c = N/T$
- $g_T(t)$ is a broadband pulse of duration $T$, BW $N/T$; i.e. it has $N$ dimensions:
  - $N$ orthogonal sequences $\{c_n\}$ can be found for multiple access (CDMA); enhanced robustness

Optimum Demodulation (preview)

- Intuitive introduction to the demodulator using the signal-space concept
  - a preview to Section 5-1
  - complete treatment requires theory of random processes
Correlation-type demodulator

- **Channel**: Additive white Gaussian noise (AWGN) is added
  \[ r(t) = s_m(t) + n(t) \]

- transmitted signal \( \{s_m(t)\}, m = 1, 2, \ldots, M \) is represented by \( N \) basis functions \( \{\psi_k(t)\}, k = 1, 2, \ldots, N \)

- received signal \( r(t) \) is projected onto these basis functions \( \{\psi_k(t)\} \)
  \[
  \int_0^T r(t)\psi_k(t)dt = \int_0^T [s_m(t) + n(t)]\psi_k(t)dt
  \]
  \[ r_k = s_{mk} + n_k, \quad k = 1, 2, \ldots, N \]

**Output vector in signal space**: \( r = s_m + n \)
Correlation demod. (cont’d)

- Received signal

\[
 r(t) = \sum_{k=1}^{N} s_{mk} \psi_k(t) + \sum_{k=1}^{N} n_k \psi_k(t) + n'(t)
\]

\[
 = \sum_{k=1}^{N} r_k \psi_k(t) + n'(t)
\]

- Correlator outputs \( r = [r_1, r_2, \ldots, r_N]^T \) are **sufficient statistik** for the decision
  - i.e.: there is **no additional info** in \( n'(t) \)
  - \( n'(t) \) is part of \( n(t) \) that is **not representable** by \( \{\psi_k(t)\} \)

- Interpretation of \( r \): **noise cloud** in signal space

3-2 Nyquist Pulse-Shaping

Filtering and pulse-shaping

- at transmitter:
  - **pulse-shaping** to reduce signal bandwidth

- at receiver
  - **filter** out noise and interferences

→ hence filtering is **applied at both sides**

- Example: Low-pass (RX) filter: introduces **inter-symbol interference (ISI)**
  - Objective is **ISI-free transmission**
  - Achieved by Nyquist filtering (e.g. root-raised-cos filter)
Nyquist Pulse-Shaping (cont’d)

Rectangular pulse at transmitter; noise added on channel; lowpass filter at receiver for noise reduction

Eye-diagram (right) shows RX signal quality

Construction of the eye-diagram

Lowpass filter introduces inter-symbol-interference (ISI)
Nyquist Pulse-Shaping (cont’d)

- PAM signal after receiver filter

\[ y(t) = \sum_{i=-\infty}^{\infty} a[i] h_e(t - iT) \]

\[ a[i] \in \{a_m\}_{m=1}^{M} \] ... PAM (or QAM) of symbol \( i \)

\( i \) ... symbol (= time) index

\( h_e(t) = g_T(t) * h_c(t) * h(t) \) ... cascade of TX pulse, channel IR, and RX filter

- Condition for ISI-free transmission

\[ h_e(kT + \tau) = \begin{cases} 
C & \text{for } k = 0 \text{ (constant)} \\
0 & \text{for } k \neq 0 
\end{cases} \]

Transmission at **minimum bandwidth** \( B_N \) (Nyquist BW)

- Assume: sampling frequency equals symbol rate

\[ f_s = 1/T \]

sampled signal can represent signals up to

\[ B_N = f_s/2 = 1/(2T) \]

- Consider a rectangular frequency response for \( H_e(f) \)

\[ H_e(f) = \text{rect}(f, B_N) \iff h_e(t) = \frac{1}{T} \text{sinc}(t/T) \]

- fulfills condition for ISI-free transmission

- but **cannot be realized** (infinite extent; not causal)
Nyquist Pulse-Shaping (cont’d)

- Cosine roll-off: allow bandwidth extension by $B_N(1 + r)$
  - frequency: convolve rectangular with cos-pulse
  - time: multiply sinc (upper) with Fourier transform of cos-pulse (center); product

left-hand figure: equivalent system impulse responses $h_e(t)$ with cos-roll-off Nyquist filtering
right-hand figure: received data sequences
Nyquist Pulse-Shaping (cont’d)

eye diagram (right-hand figure):
- (long) sequence of received, filtered data symbols
- superimposed in diagram over two symbol intervals