Fund. of Digital Communications
Ch. 3: Digital Modulation

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Outline

- 3-1 Pulse Amplitude Modulation
  - Baseband and Bandpass Signals
  - One-, Two-, and Multidimensional Signals
  - QAM and Complex Equivalent Baseband Signals
- 3-2 Pulse Shaping and ISI-free Transmission
  - Signal Spectrum
  - Nyquist Pulse Shaping

References and Figures

- Figures refer to Chapter 7 of
  - J. G. Proakis and M. Salehi, “Communication System Engineering,” 2nd Ed., Prentice Hall, 2002
  - J. G. Proakis and M. Salehi, “Grundlagen der Kommunikationstechnik,” 2. Aufl., Pearson, 2004 (in German)
- References to figures denoted as [7.x]

3-1 Pulse Amplitude Modulation (PAM)

- In practice: TX signal is a stream of symbols

$$s(t) = \sum_{i=-\infty}^{\infty} s^{[i]}(t - iT)$$

where $s^{[i]}(t) \in \{s_m(t)\}_{m=1}^{M}$ represents symbol $s[i]$, taken from an M-ary alphabet $\{s_m(t)\}$

- Basic assumption:
  - Consecutive symbols **do not interfere**
  - Thus we can concentrate on **one single symbol**
  - Symbol index i is dropped “without loss of generality”; the TX signal is $s(t) \in \{s_m(t)\}_{m=1}^{M}$

Pulse Amplitude Modulation (PAM)

- Transmission of information through **modulation of signal amplitude**
  - Signal shape $g_T(t)$ is tailored to channel
- **Baseband signals** – for baseband channels [7.4]
  - Binary antipodal modulation;
    selects amplitude of a pulse waveform $g_T(t)$

$$\begin{aligned} \text{“1”} &\hat{=} \quad A \quad \text{for } s_1(t) = Ag_T(t) \\ \text{“0”} &\hat{=} \quad -A \quad \text{for } s_2(t) = -Ag_T(t) \end{aligned}$$

  - Bit rate R_b, bit interval T_b (= symbol interval)

$$R_b = \frac{1}{T_b}$$

PAM, Baseband (cont'd)

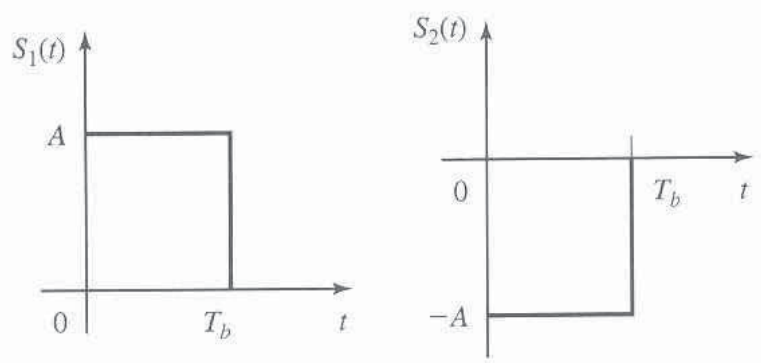


Figure 7.4 Binary PAM signals.

PAM, Baseband (cont'd)

- M-ary PAM [7.5]
  - Usually: $M = 2^k$ for integer k; k ... nb. bits/symbol
  - Symbol interval: $T = k/R_b = kT_b$ [7.6]
- (Set of) M signal waveforms [7.7]

$$\boxed{s_m(t) = A_m g_T(t)}, \text{ for } m \in \{1, 2, ..., M\}, 0 \leq t \leq T$$

  - Pulse shape $g_T(t)$ determines **signal spectrum** [7.9]
  - Energy (can vary for $m \in \{1, 2, ..., M\}$)

$$\mathcal{E}_m = \int_0^T s_m^2(t)dt = A_m^2 \int_0^T g_T^2(t)dt = A_m^2 \mathcal{E}_g$$

$\mathcal{E}_g$ ... energy of pulse $g_T(t)$

PAM, Baseband (cont'd)

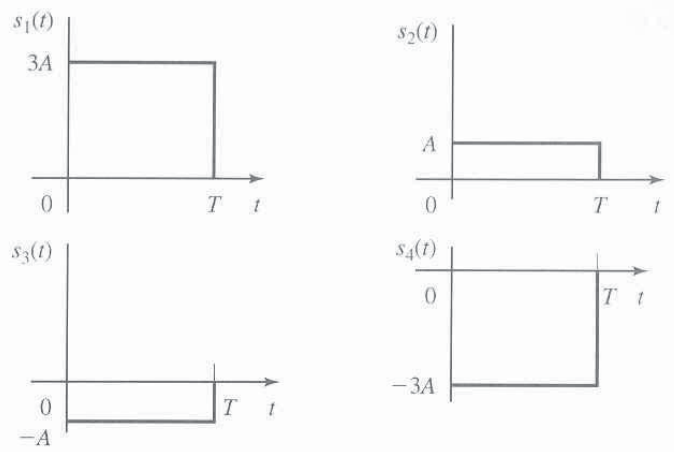
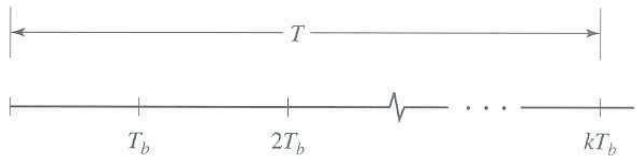


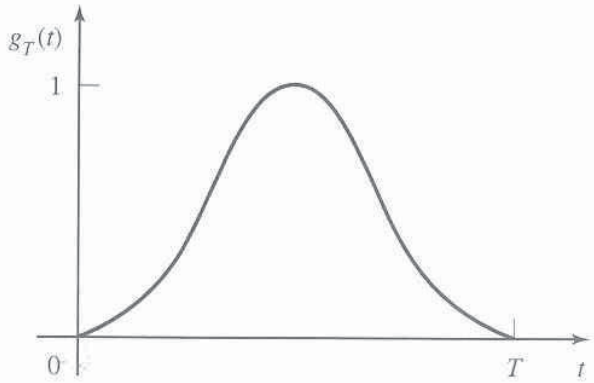
Figure 7.5 $M = 4$ PAM signal waveforms.



T_b = bit interval
T = symbol interval

Figure 7.6 Relationship between the symbol interval and the bit interval.

PAM, Baseband (cont'd)



(a)

Figure 7.7 Signal pulse for PAM.

PAM, Bandpass (Passband)

- **Bandpass signals** – for bandpass channels
- **Carrier modulation** [7.8]
  - Multiplication of $s_m(t)$ by carrier $\cos(2\pi f_c t)$

$$u_m(t) = s_m(t)\cos(2\pi f_c t) = A_m g_T(t)\cos(2\pi f_c t),$$

    for $m \in \{1, 2, ..., M\}$,
    f_c ... carrier frequency (center frequency)
  - **in frequency domain** [7.9]:

$$U_m(f) = \frac{A_m}{2}[G_T(f - f_c) + G_T(f + f_c)]$$

  - DSB-SC-AM (Dual sideband, suppressed carr. AM)
  - Channel bandwidth $2W$ (**doubled** w.r.t. baseband!)

PAM, Baseband (cont'd)

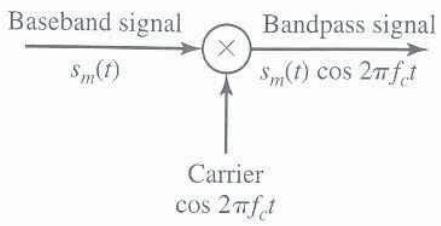


Figure 7.8 Amplitude modules sinusoidal carrier by the baseband signal.

PAM, Baseband (cont'd)

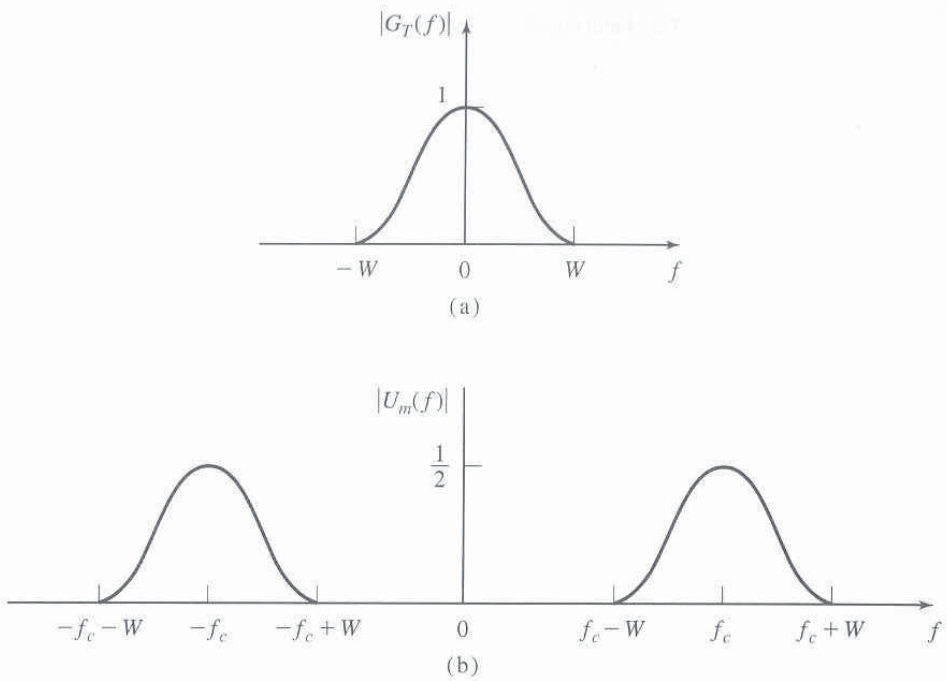


Figure 7.9 Spectra of (a) baseband and (b) amplitude-modulated signals.

Definitions of Bandwidth

- Definitions
  - Absolute bandwidth
  - 3-dB bandwidth
  - equivalent bandwidth (BW of **block spectrum** with equal energy and const. amplitude as at f_c)
  - first spectral zero (BW of main lobe)
- **Time-bandwidth product** is **constant**!
  - e.g. first zero of rectangular pulse:
  - **Interval** T vs. **first zero** B_z of its Fourier transform:

$$B_z = 1/T, \text{ hence } \boxed{TB_z = 1}$$

Geometric Representation in Signal Space

- PAM Signals are **one-dimensional**

$$s_m(t) = s_m \psi(t)$$

- baseband:

$$\begin{aligned} \psi(t) &= \frac{1}{\sqrt{\mathcal{E}_g}} g_T(t), \quad 0 \le t \le T \\ s_m &= \sqrt{\mathcal{E}_g} A_m, \quad m \in \{1, 2, ..., M\} \end{aligned}$$

- bandpass:

$$\begin{aligned} \psi(t) &= \sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \cos 2\pi f_c t \\ s_m &= \sqrt{\mathcal{E}_g/2} A_m, \quad m \in \{1, 2, ..., M\} \end{aligned}$$

Geometric Representation (cont'd)

- Euclidean distance

$$d_{mn} = \sqrt{|s_m - s_n|^2}$$

- Energy of PAM signals (baseband)

$$\mathcal{E}_m = s_m^2 = \mathcal{E}_g A_m^2, \quad m \in \{1, 2, ...M\}$$

- e.g.: symmetric PAM [7.11]

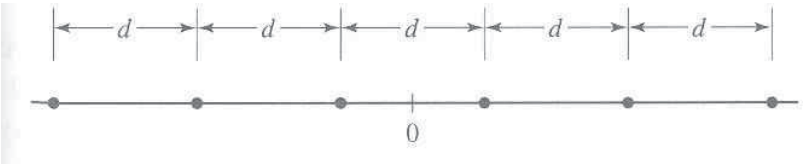


Figure 7.11 Signal points (constellation) for symmetric PAM.

Two- (and Multidimensional) Signals

- **Simultaneous PAM** of two (or more) basis functions
  - yields additional points in N-dim. signal space; each representing a signal waveform
- **orthogonal** signals
  - M-ary symbols are represented by $N = M$ orthogonal waveforms
  - (see [7.12]–[7.14] for $M = 2$)

Two-dimensional Signals

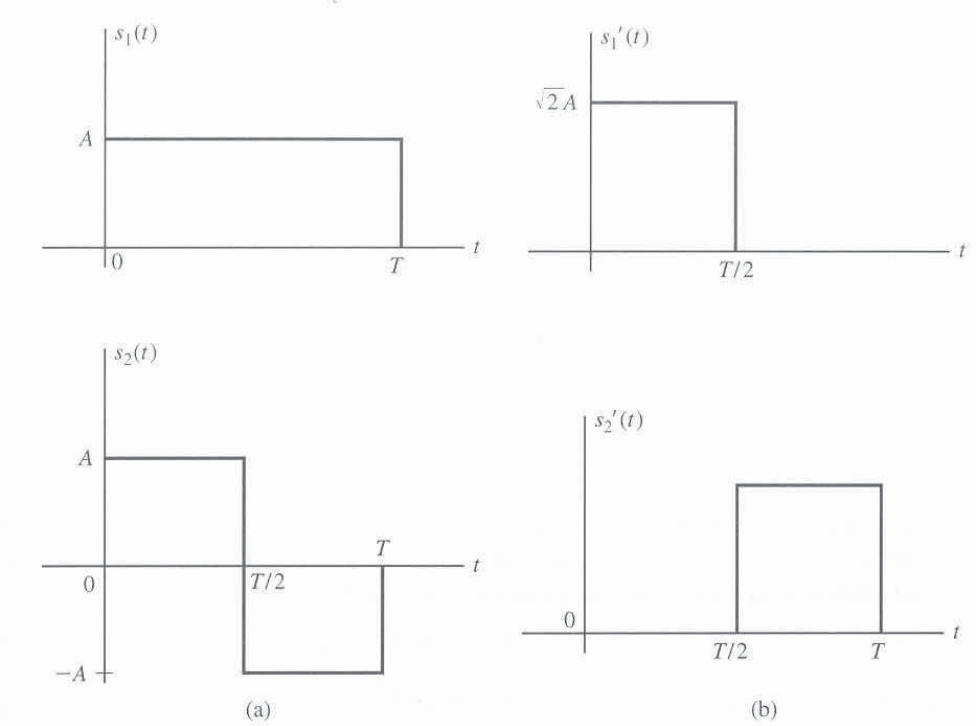


Figure 7.12 Two sets of orthogonal signals.

Two-dimensional Signals

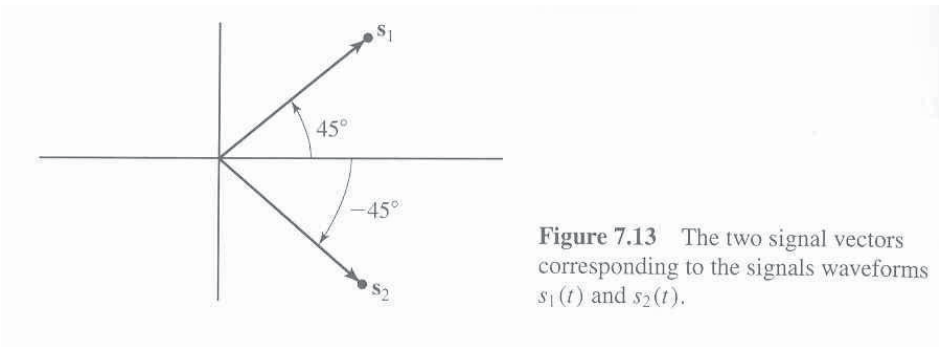


Figure 7.13 The two signal vectors corresponding to the signals waveforms $s_1(t)$ and $s_2(t)$.

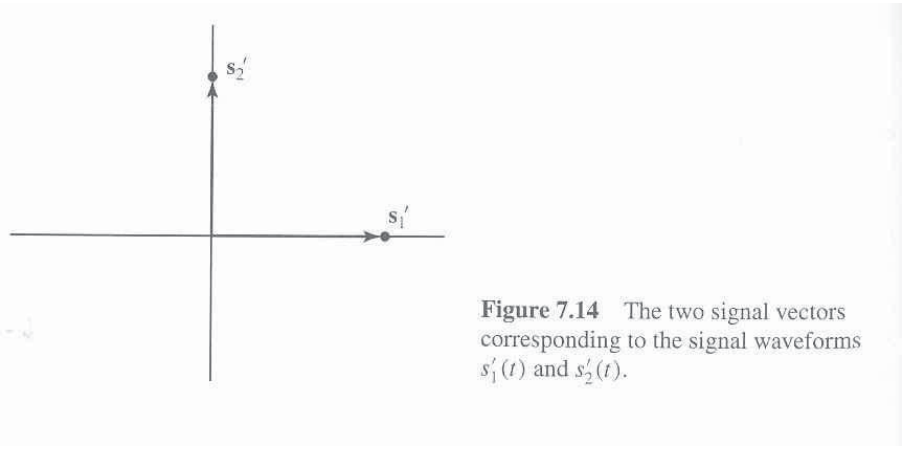


Figure 7.14 The two signal vectors corresponding to the signal waveforms $s_1'(t)$ and $s_2'(t)$.

Two-dimensional Signals

- **bi-orthogonal** signals [7.15]
  - **binary antipodal PAM** of the basis functions
  - $M = 4$-ary signals with equal energies
  - add signal vectors with inverted polarities:

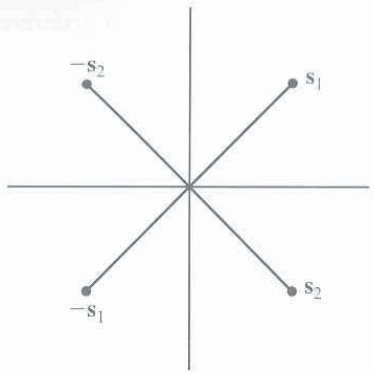


Figure 7.15 Signal constellation for $M = 4$ biorthogonal signals.

Two-dimensional Signals

- $M = 8$-ary signals with equal energies

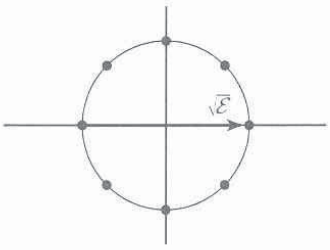


Figure 7.16 $M = 8$ signal-point constellation corresponding to the two points of orthogonal signal waveforms in Figure 7.12 and their negatives, i.e., $s_1(t)$, $s_2(t)$, $s_1'(t)$, $s_2'(t)$, $-s_1(t)$, $-s_2(t)$, $-s_1'(t)$ and $-s_2'(t)$.

- $M = 8$-ary signals with (two) different energies

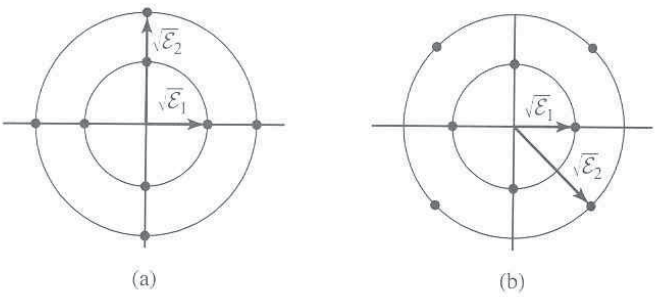


Figure 7.17 Two $M = 8$ signal-point constellations in two dimensions, corresponding to a superposition of two sets of biorthogonal signal waveforms with different energies.

Two-dimensional Bandpass Signals–QAM

- important example for 2D-signals: (digital) **QAM**
  - PAM of the orthogonal carriers

$$u_m(t) = A_{mc} g_T(t) \cos 2\pi f_c t - A_{ms} g_T(t) \sin 2\pi f_c t$$

  - in geometric representation:

$$u_m(t) = s_{m1}\psi_1(t) + s_{m2}\psi_2(t), \text{ with}$$

$$\psi_1(t) = \sqrt{2/\mathcal{E}_g}\, g_T(t) \cos 2\pi f_c t$$

$$\psi_2(t) = -\sqrt{2/\mathcal{E}_g}\, g_T(t) \sin 2\pi f_c t$$

$$\mathbf{s}_m = [s_{m1}, s_{m2}]^T = [\sqrt{\mathcal{E}_s}\, A_{mc}, \sqrt{\mathcal{E}_s}\, A_{ms}]^T$$

- QAM signals have a **complex-valued** equivalent baseband representation; **no BW loss!** (see Chapter 2)

Two-dimensional QAM Signals

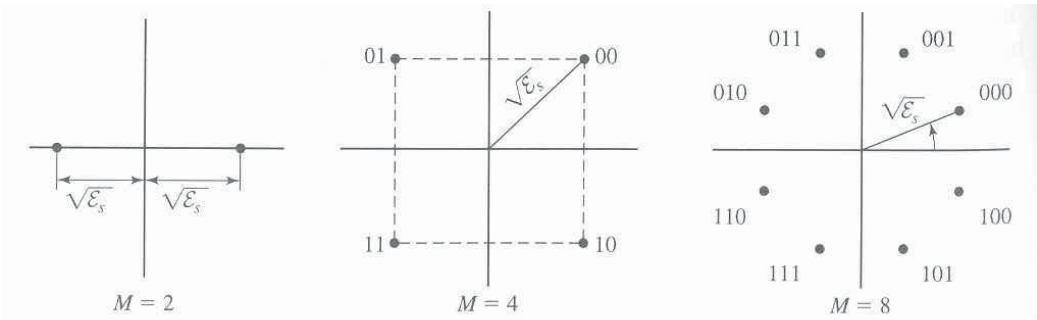


Figure 7.20 PSK signal constellations.

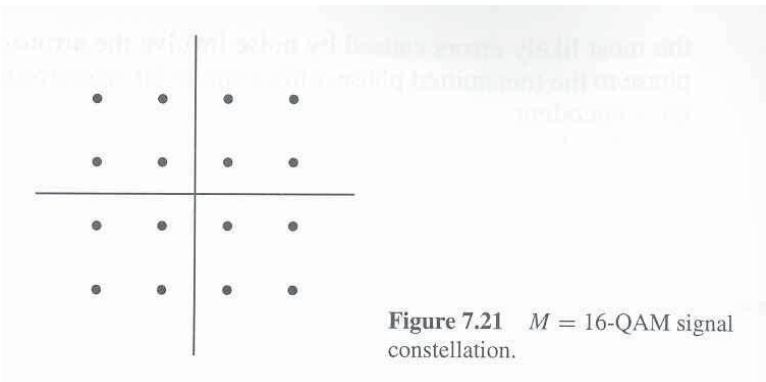


Figure 7.21 $M = 16$-QAM signal constellation.

Two-dimensional Bandpass Signals–QAM (cont'd)

- Functional block diagram of a (digital) QAM modulator

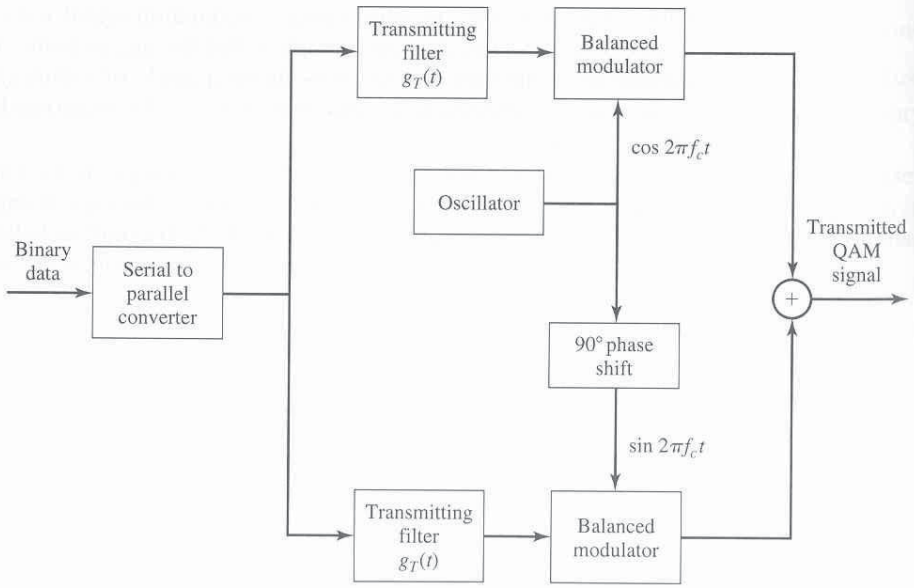


Figure 7.22 Functional block diagram of modulator for QAM.

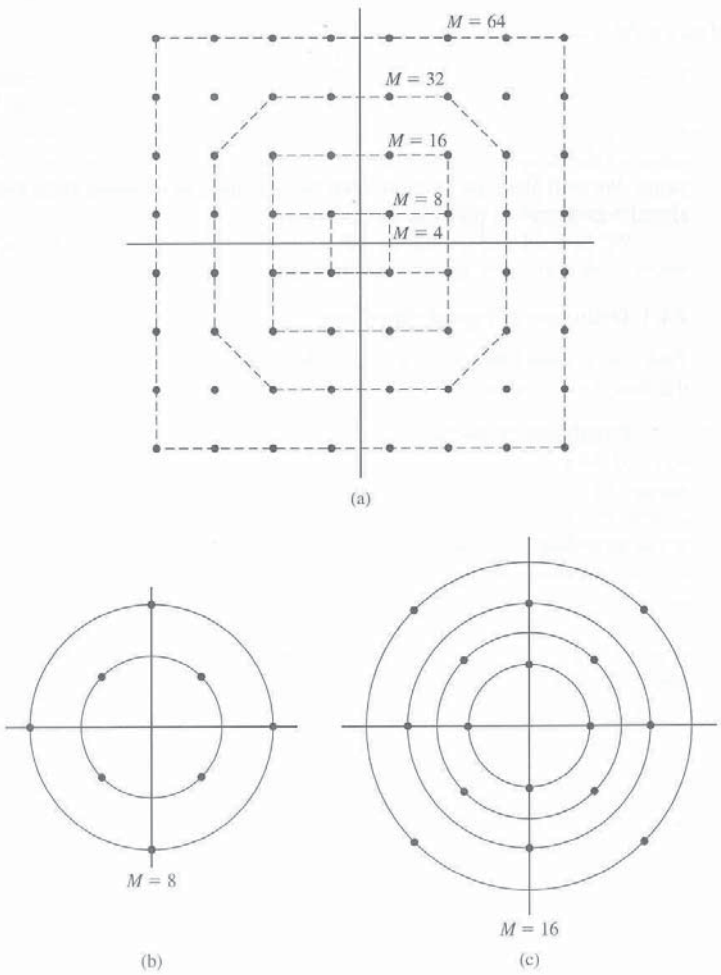


Figure 7.23 (a) Rectangular signal-space constellations for QAM. (b, c) Examples of combined PAM-PSK signal-space constellations.

Multidimensional Signals

- Two-dimensional case: $M = 2^k$ signals have been constructed in 2D
- Multidimensional case: (OPAM)
  - construct N orthogonal signals
  - define signal points in these dimensions

$$s_\mu(t) = \sum_{n=1}^{N} a_{m,n} g_n(t)$$

$\{g_n(t)\}_{n=1}^{N}$	...	N orthogonal waveforms (basis)
$\{a_{m,n} \in \mathbb{R}(\text{or } \mathbb{C})\}$	...	PAM (or QAM) symbols (M-ary) for n-th waveform; $m \in 1, 2, \ldots, M$
$\{s_\mu(t)\}_{\mu=1}^{MN}$	...	MN-ary set of waveforms

Multidimensional Signals

- **OFDM**: parallel transmission to enlarge symbol duration against inter-symbol-interference (ISI)

$$g_n(t) = \frac{1}{\sqrt{T}} e^{j2\pi nt/T} w(t) \quad \ldots \quad n\text{-th subcarrier at } f = n/T$$

$$s_\mu(t) = \frac{1}{\sqrt{T}} \sum_{n=-N/2}^{N/2-1} a_{m,n} e^{j2\pi nt/T} w(t)$$

$\{g_n(t)\}$	...	orthogonal subcarriers (Fourier basis)
$w(t)$	...	window function (e.g. rectangular)
$\{a_{m,n} \in \mathbb{C}\}$	...	QAM symbols (e.g. QPSK, 16/64-QAM)

- the symbols $s_\mu(t)$ are MN-ary (N subcarriers)

Multidimensional Signals

- **Spread Spectrum**/CDMA: few points in $N \gg 1$ dimensional space (large TB-product)
  - divide T into N chip intervals $T_c = T/N$
  - modulate chip waveform with "spreading code" $\{c_n\}$

$$g_T(t) = \sum_{n=0}^{N-1} c_n g_c(t - nT_c); \quad s_m(t) = a_m g_T(t) \text{ (PAM/QAM)}$$

  - chip wavef. $g_c(t)$ has N-fold bandwidth $\propto 1/T_c = N/T$
  - $g_T(t)$ is a broadband pulse of duration T, BW N/T; i.e. it has N dimensions:
  - N orthogonal sequences $\{c_n\}$ can be found for multiple access (CDMA); enhanced robustness

Optimum Demodulation (preview)

- Intuitive introduction to the *de*modulator using the signal-space concept
  - a **preview** to Section 5-1
  - complete treatment requires theory of **random processes**

Correlation-type demodulator

- **Channel:** Additive white Gaussian noise (AWGN) is added

$$r(t) = s_m(t) + n(t)$$

- transmitted signal $\{s_m(t)\}, m = 1, 2, ...M$ is represented by N basis functions $\{\psi_k(t)\}, k = 1, 2, ...N$
- received signal $r(t)$ is projected onto these basis functions $\{\psi_k(t)\}$

$$\int_0^T r(t)\psi_k(t)dt = \int_0^T [s_m(t) + n(t)]\psi_k(t)\, dt$$

$$r_k = s_{mk} + n_k, \quad k = 1, 2, ..., N$$

Correlation demod. (cont'd)

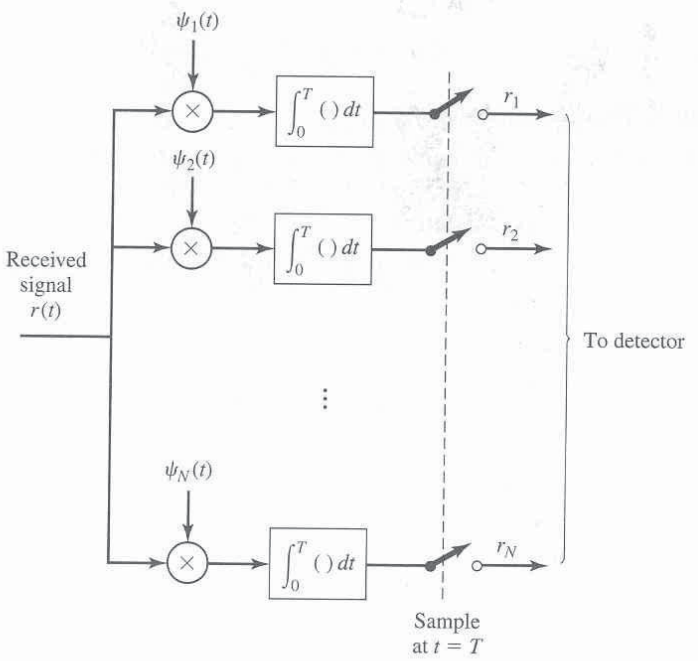


Figure 7.31 Correlation-type demodulator.

[Proakis 2002]

- Output vector in signal space: $\mathbf{r} = \mathbf{s}_m + \mathbf{n}$

Correlation demod. (cont'd)

- Received signal

$$r(t) = \sum_{k=1}^{N} s_{mk}\psi_k(t) + \sum_{k=1}^{N} n_k\psi_k(t) + n'(t)$$
$$= \sum_{k=1}^{N} r_k\psi_k(t) + n'(t)$$

- Correlator outputs $\mathbf{r} = [r_1, r_2, ...r_N]^T$ are **sufficient statistik** for the decision
  - i.e.: there is **no additional info** in $n'(t)$
  - $n'(t)$ is part of $n(t)$ that is **not representable** by $\{\psi_k(t)\}$
- Interpretation of $\mathbf{r}$: **noise cloud** in signal space

3-2 Nyquist Pulse-Shaping

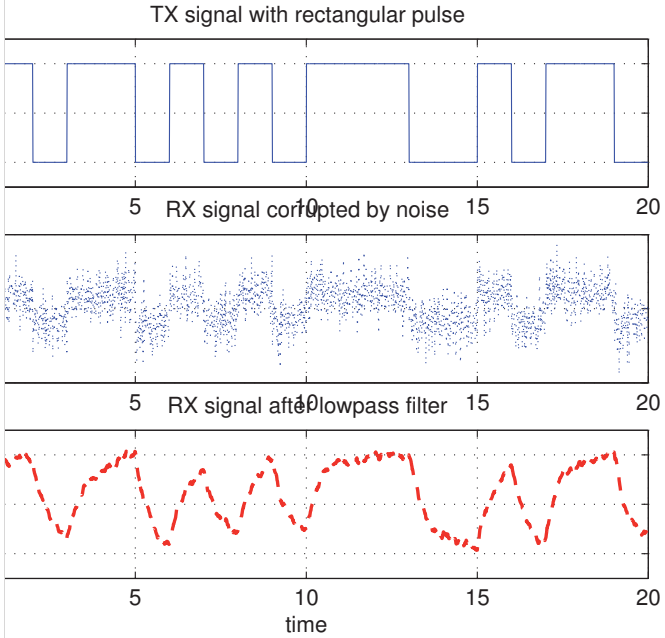
Filtering and pulse-shaping

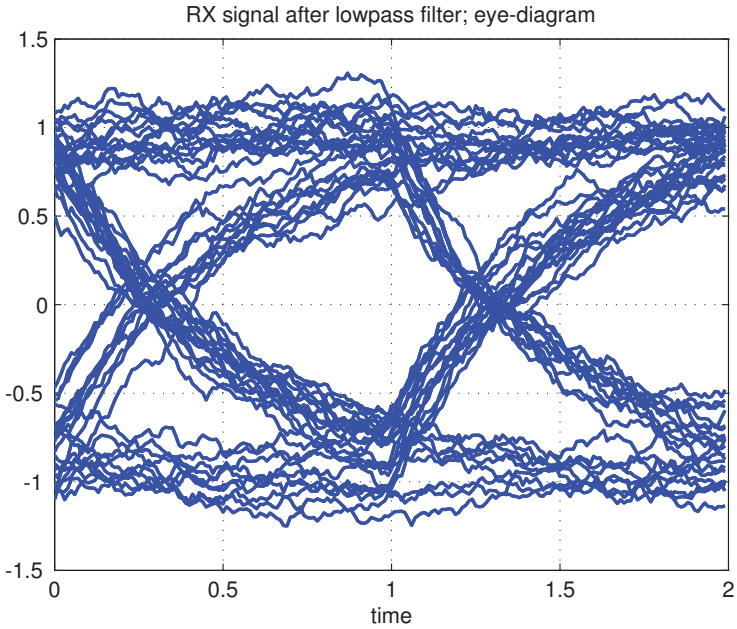
- at transmitter:
  - **pulse-shaping** to reduce signal bandwith
- at receiver
  - **filter** out noise and interferences

→ hence filtering is **applied at both sides**

- Example: Low-pass (RX) filter: introduces **inter-symbol interference (ISI)**
  - Objective is **ISI-free transmission**
  - Achieved by Nyquist filtering (e.g. root-raised-cos filter)

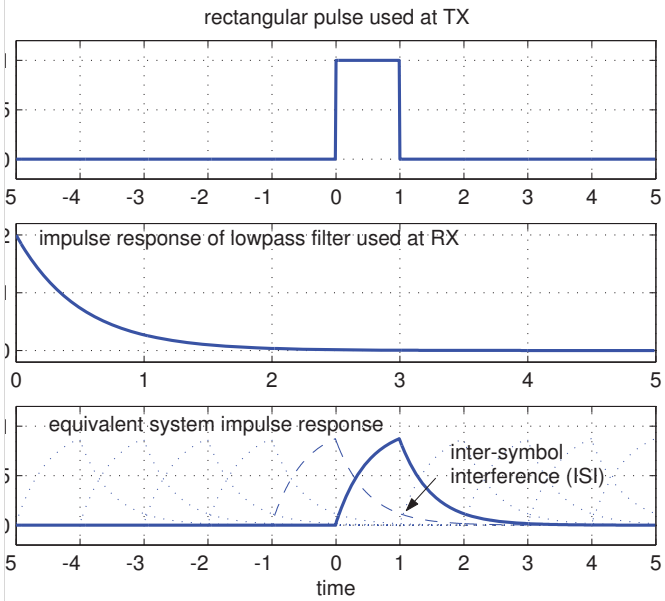
Nyquist Pulse-Shaping (cont'd)

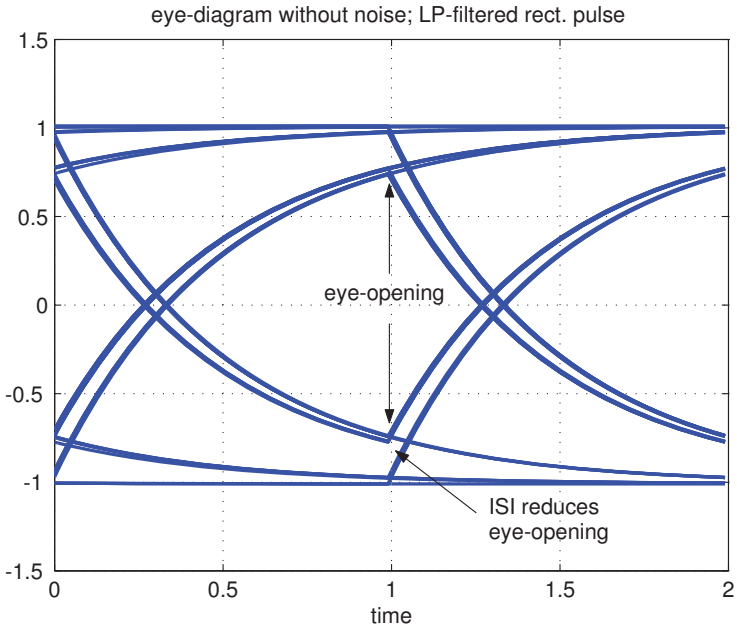




- **Rectangular pulse** at transmitter; **noise** added on channel; **lowpass filter** at receiver for noise reduction
  - **Eye-diagram** (right) shows RX signal quality

Nyquist Pulse-Shaping (cont'd)





- Construction of the eye-diagram
  - Lowpass filter introduces **inter-symbol-interference** (ISI)

Nyquist Pulse-Shaping (cont'd)

- PAM signal after receiver filter

$$y(t) = \sum_{i=-\infty}^{\infty} a[i] h_e(t - iT)$$

$a[i] \in \{a_m\}_{m=1}^{M}$ ... PAM (or QAM) of symbol i
i ... symbol (= time) index
$h_e(t) = g_T(t) * h_c(t) * h(t)$ ... cascade of TX pulse, channel IR, and RX filter

- Condition for ISI-free transmission

$$h_e(kT + \tau) = \begin{cases} C & \text{for } k = 0 \text{ (constant)} \\ 0 & \text{for } k \neq 0 \end{cases}$$

Nyquist Pulse-Shaping (cont'd)

Transmission at **minimum bandwidth** B_N (Nyquist BW)

- Assume: sampling frequency equals symbol rate

$$f_s = 1/T$$

sampled signal can represent signals up to

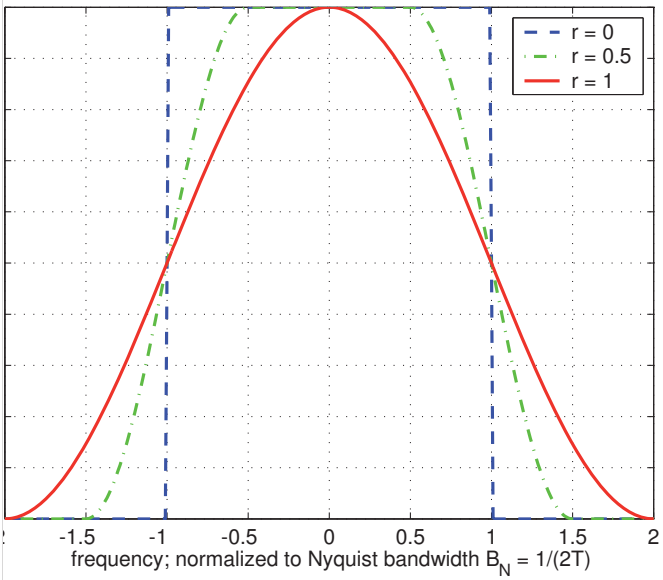
$$B_N = f_s/2 = 1/(2T)$$

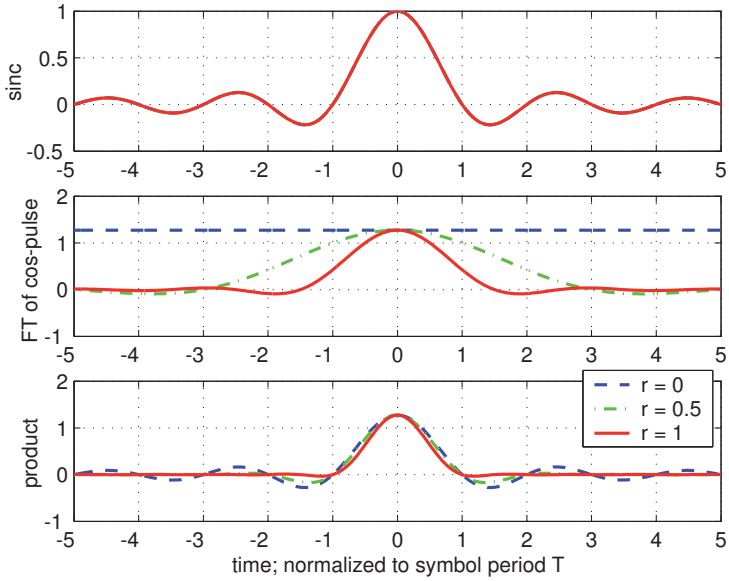
- Consider a rectangular frequency response for $H_e(f)$

$$H_e(f) = \text{rect}(f, B_N) \overset{\mathcal{F}}{\longleftrightarrow} h_e(t) = \frac{1}{T}\text{sinc}(t/T)$$

  - fulfills condition for ISI-free transmission
  - but **cannot be realized** (infinite extent; not causal)

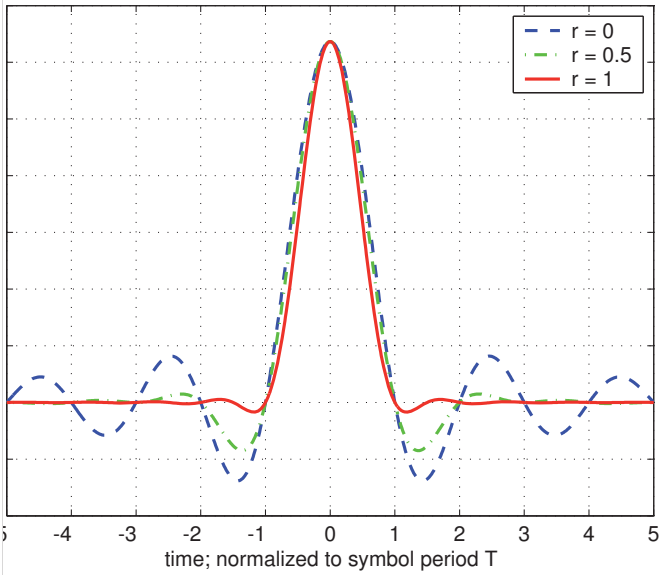
Nyquist Pulse-Shaping (cont'd)

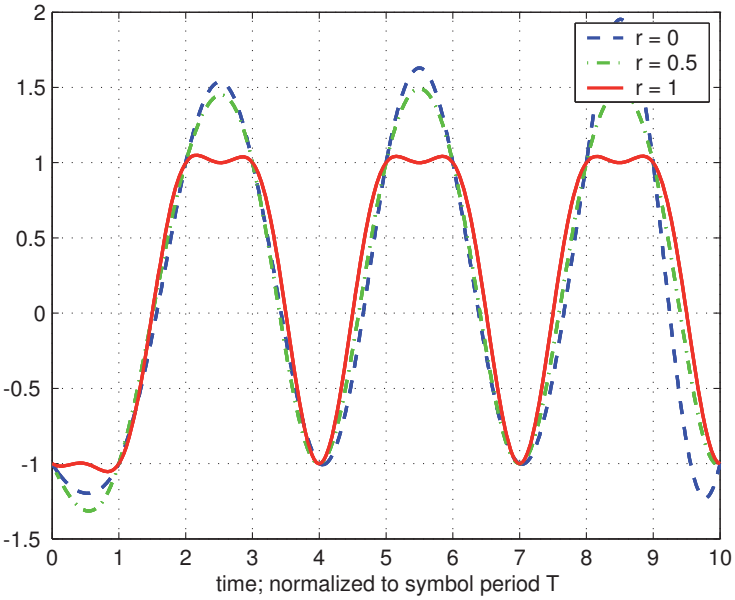




- Cosine roll-off: allow bandwidth extension by $B_N(1+r)$
  - frequency: convolve rectangular with $\cos$-pulse
  - time: multiply sinc (upper) with Fourier transform of $\cos$-pulse (center); product

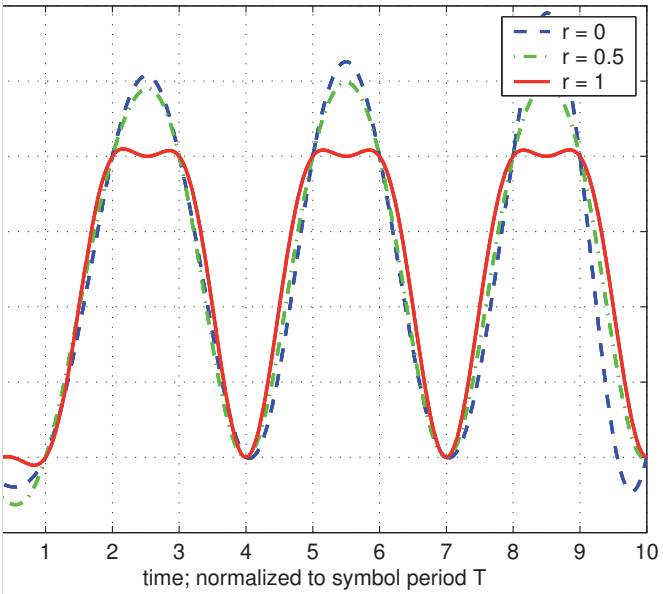
Nyquist Pulse-Shaping (cont'd)

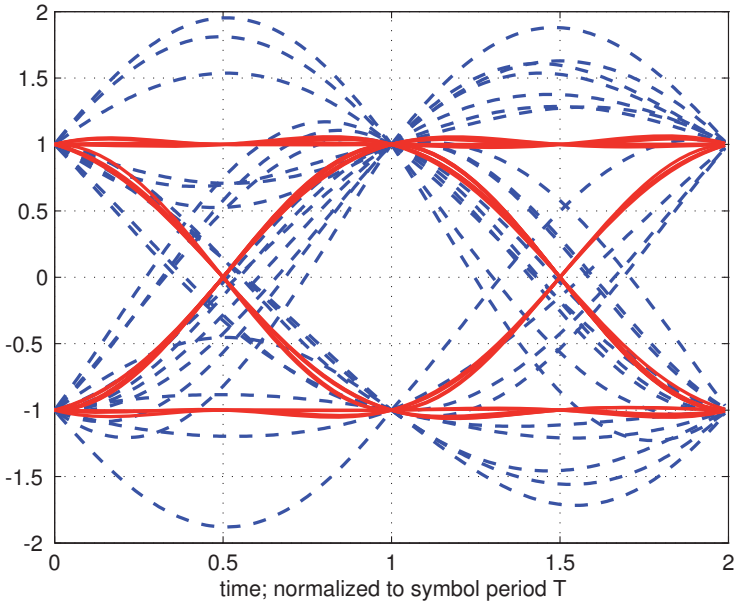




- left-hand figure: equivalent system impulse responses $h_e(t)$ with $\cos$-roll-off Nyquist filtering
- right-hand figure: received data sequences

Nyquist Pulse-Shaping (cont'd)





eye diagram (right-hand figure):

- (long) sequence of received, filtered data symbols
- superimposed in diagram over two symbol intervals