Mobile Radio Systems –
Small-Scale Channel Modeling

Klaus Witrisal
witrisal@tugraz.at

Signal Processing and Speech Communication Laboratory
www.spsc.tugraz.at
Graz University of Technology

October 28, 2015

Outline

■ 3-1 Introduction – Mathematical models for communications channels [Molisch 6.2.2; Proakis 1-3]
■ 3-2 Stochastic Modeling of Fading Multipath Channels
  ◆ Multipath channel [Proakis 14-1]
  ◆ Fading amplitude distribution (Rayleigh, Rice) [Molisch 5.4, 5.5]
  ◆ Time-selective fading [Molisch 5.6]
  ◆ Frequency-selective fading
  ◆ WSSUS stochastic channel description [Molisch 6.3-6.5, Proakis 14]
■ 3-3 Classification of Small-Scale Fading [Molisch 6.5]
References

- A. F. Molisch: *Wireless Communications*, 2005, Wiley
- M. Pätzold: *Mobile Fading Channels*, 2002, Wiley

Figures (partly) extracted from these references

Signal Models

- “Signal processing” channel models can be described for different interfaces
- Application/design objective determines choice of appropriate model
Additive Noise Channel

Channel’s frequency response is **flat** over signal bandwidth

- Simplest model – transmitted (TX) signal corrupted by additive noise
  \[ r(t) = \alpha s(t) + n'(t) \]

- \( s(t) \) ... TX signal
  - is a **bandpass signal** \( s(t) = \sqrt{2} \Re \{ s_i(t) e^{j2\pi f_c t} \} \)

- \( r(t) \) ... received (RX) signal

- for (lowpass equivalent) baseband signals (i.e. complex envelopes of \( s(t), r(t), n'(t) \))
  \[ r_l(t) = h s_l(t) + n'_l(t), \quad \text{with } h \in \mathbb{C} \]

Additive Noise Channel (cont’d)

- Noise is usually modeled as white, Gaussian (additive white Gaussian noise – AWGN)
  \[ \phi_{n'}(\tau) = \mathbb{E}\{n'(t)n'(t+\tau)\} = \frac{N_0}{2} \delta(\tau) \quad \xrightarrow{\mathcal{F}} \quad S_{n'}(f) = \frac{N_0}{2} \]
Additive Noise Channel (cont’d)

- Sampled AWGN model (lowpass equivalent model)
  \[ r[k] = h s[k] + n[k] \] (all are \( \in \mathbb{C} \))

- Noise characterization
  \[ \mathbb{E}\{n[k]n^*[l]\} = \sigma_n^2 \delta[k-l] \]

  - \( n[k] \) is zero-mean circularly symmetric complex Gaussian (ZMCSCG)
  - Real and imaginary components are i.i.d.
    (independent, identically distributed)
  - \( \sigma_n^2 \) depends on (matched) filter at receiver
    front-end
  - Real and imaginary components have \( \sigma_n^2/2 \)

Linear filter channel

Channel’s frequency response is frequency-selective (i.e. non-flat), leading to (linear) signal distortions

- For time-invariant channels
  \[ r(t) = s(t) * c(t) + n(t) \]
  \[ = \int_{-\infty}^{\infty} c(\tau)s(t-\tau)d\tau + n(t) \]

- \( c(t) \) ... impulse response of linear filter
Linear filter channel (cont’d)

- Sampled case (lowpass equivalent model)

\[ r[k] = \sum_{l=0}^{L-1} h[l] s[k - l] + n[k] \]

- \( h[k] \) incorporates
  - TX pulse shape
  - RX (matched) filter; ADC filter
  - (thus bandwidth corresponds to signal bandwidth)
  - physical channel

- \( n[k] \) ... AWGN (ZMCSCG)

This is actually an equivalent, whitened matched filter (WMF) channel model [Barry/Lee/Messerschmitt]

Linear time-variant filter channel

- Characterized by time-variant channel impulse response (CIR) \( c(\tau; t) \)
  - response of channel at time \( t \)
  - to an impulse transmitted at time \( t - \tau \)

- \( \tau \) ... “elapsed time”, “age” variable

\[
\begin{align*}
    r(t) &= s(t) * c(\tau; t) + n(t) \\
    &= \int_{-\infty}^{\infty} c(\tau; t)s(t - \tau)d\tau + n(t)
\end{align*}
\]

- model for multipath propagation

\[
c(\tau; t) = \alpha_i(t) \delta(\tau - \tau_i(t)) \quad (1)
\]
Stochastic modeling of fading multipath channels

- Motivated by their **randomly time-variant** nature and **large number** of multipath components

- Derivation of **lowpass equivalent** CIR from (1)

\[
c_l(\tau; t) = \sum_{i=0}^{\infty} \alpha_i(t)e^{-j2\pi f_c \tau_i(t)} \delta(\tau - \tau_i(t))
\]

\[
= \sum_{i=0}^{\infty} \alpha_i(t)e^{j\varphi_i(t)} \delta(\tau - \tau_i(t))
\]

(2)

considering discrete multipath components

- phase term \( \varphi_i(t) = -2\pi f_c \tau_i(t) \) **varies** dramatically
Fading of an unmodulated carrier

- TX signal is unmodulated carrier (CW) $s_l(t) = 1$
- RX signal w/o noise: $y_l(t) = c_l(\tau; t) \ast 1 = c_l(t) \cdot 1$

$$c_l(t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)} = \sum_{i=0}^{\infty} \alpha_i(t) e^{-j2\pi f_c \tau_i(t)}$$

sum of vectors (phasors)
- amplitudes $\alpha_i(t)$ change slowly
- phases $\varphi_i(t)$ change by $2\pi$ if:
  - $\tau_i(t)$ changes by $1/f_c$
  - i.e.: path length changes by wavelength $\lambda$
- large number of multipath components

→ model $c_l(t)$ as a random process!

Fading of an unmodulated carrier (cont’d)

Modeling $c_l(t)$ as a random process:
- large number of multipath components are added
- by central limit theorem (CLT):
  - $c_l(t)$ is complex Gaussian
  - (CIR $c_l(\tau; t)$ is complex Gaussian)
- $c_l(t)$ has random phase and amplitude

- in **absence** of dominant component:
  - $c_l(t)$ is zero-mean complex Gaussian

→ its envelope $|c_l(t)|$ is Rayleigh distributed
- Rayleigh fading channel
Fading of an unmodulated carrier (cont’d)

Rayleigh distribution:

\[ f_R(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)} \text{ for } r \geq 0 \]

characterized by \( \sigma^2 \): variance of underlying Gaussian processes \( X_1, X_2 \sim \mathcal{N}(0, \sigma^2) \), where \( X_1 = \Re\{c(t)\} \) and \( X_2 = \Im\{c(t)\} \)

derivation of Rayleigh distribution ...

- \( Y = X_1^2 + X_2^2 \) ... has \( \chi^2 \)-PDF of 2 degrees of freedom
- \( R = \sqrt{X_1^2 + X_2^2} \) ... amplitude \( |c(t)| \) has Rayleigh PDF
Fading of an unmodulated carrier (cont’d)

- in presence of a dominant component: 
  \( c_l(t) \) is non-zero-mean complex Gaussian

→ its envelope \(|c_l(t)|\) is Ricean distributed

- Ricean fading channel

- Ricean distribution:
  \[
  f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+s^2}{2\sigma^2}} I_0 \left( \frac{rs}{\sigma^2} \right) \quad \text{for } r \geq 0
  \]

  \( I_0(x) \) ... zero-order modified Bessel function of first kind

- characterized by
  \( \sigma^2 \) ... variance of underlying Gaussian processes and
  \( s^2 = m_1^2 + m_2^2 \) ... power of mean (i.e. \( s^2 = |E\{c_l(t)\}|^2 \))
Fading of an unmodulated carrier (cont’d)

- Shape of Ricean distribution defined by

\[ K = \frac{s^2}{2\sigma^2} \]

\[ K \ [dB] = 10 \log \frac{s^2}{2\sigma^2} \]

**Ricean \( K \)-factor**

- Ratio of deterministic signal power (mean) and variance of multipath (scattered components)
- For \( K = 0 = -\infty \) dB:
  Ricean distribution equivalent to Rayleigh

Ricean (and Rayleigh) PDFs
Ricean (and Rayleigh) CDFs
Time-selective fading

- Characterization of the time variability

\[ s_l(t) = 1 \quad \rightarrow \quad y_l(t) = c_l(\tau; t) \ast 1 = c_l(t) = \sum_{i=0}^{\infty} \alpha_i(t)e^{j\varphi_i(t)} \]

- Characterize autocorrelation function of \( c_l(t) \)
  - assume \( c_l(t) \) is complex Gaussian
  - assume \( c_l(t) \) is wide-sense stationary (WSS)

- Define: spaced-time correlation function

\[ \phi_c(\Delta t) = E\{c_l^*(t)c_l(t+\Delta t)\} \quad \leftrightarrow \quad S_c(\nu) \]

- Doppler power spectrum \( S_c(\nu) = \int_{-\infty}^{\infty} \phi_c(\Delta t)e^{-j2\pi\nu\Delta t}d\Delta t \)

Doppler power spectrum: average power output of channel as a function of Doppler frequency
Time-selective fading (cont’d)

Jakes model for Doppler power spectrum

- assumes mobile moving at const. velocity $v$
- uniformly distributed scattering around mobile
- Jakes Doppler spectrum:

$$S_c(\nu) = \frac{1}{\pi} \frac{1}{\sqrt{\nu_{\text{max}}^2 - \nu^2}}$$

(for normalized power)
Time-selective fading (cont’d)

Characterization of time-selective fading by parameters

- RMS Doppler spread
  \[ \nu_{\text{rms}} = \sqrt{\nu^2 - \bar{\nu}^2} \]
  second centralized moment of normalized Doppler PSD

- mean and mean squared Doppler spread
  \[
  \begin{align*}
  \bar{\nu} &= \frac{\int \nu S_c(\nu) d\nu}{\int S_c(\nu) d\nu} \\
  \bar{\nu}^2 &= \frac{\int \nu^2 S_c(\nu) d\nu}{\int S_c(\nu) d\nu}
  \end{align*}
  \]

- Coherence time
  \[ T_c \approx \frac{1}{\nu_{\text{rms}}} \]

Frequency-selective fading

for a (time-invariant) multipath channel

- Characterization of the time dispersion: CIR \( c_l(\tau) \)
  \[
  s_l(t) = \delta(t) \quad \rightarrow \quad y_l(t) = c_l(\tau; t) \ast \delta(t) = \sum_{i=0}^{\infty} \alpha_i(t) e^{j\varphi_i(t)} \delta(t - \tau_i(t))
  \]
  \[
  c_l(\tau) = \sum_{i=0}^{\infty} \alpha_i e^{j\varphi_i} \delta(\tau - \tau_i)
  \]

- ACF: Uncorrelated scattering assumption:
  \[
  E\{c_l^*(\tau_1)c_l(\tau_2)\} = S_c(\tau_1)\delta(\tau_1 - \tau_2)
  \]

\( S_c(\tau) \ldots \text{multipath intensity profile} (= \text{delay power spectrum}; = \text{average power delay profile}) \)
Frequency-selective fading (cont’d)

- Time-dispersion implies frequency-selectivity
  - Equivalent channel characterization by **channel transfer function** (TF) $C_l(f)$
    \[
    c_l(\tau) \overset{F}{\leftrightarrow} C_l(f) = \int_{-\infty}^{\infty} c_l(\tau) e^{-j2\pi f \tau} d\tau
    \]

- ACF of channel TF
  \[
  S_c(\tau) \overset{F}{\leftrightarrow} \phi_C(\Delta f) = E\{C_l^*(f)C_l(f + \Delta f)\}
  \]
  $\phi_C(\Delta f)$ ... spaced-frequency correlation function
  - TF $C_l(f)$ is **wide-sense stationary** (WSS in $f$) if CIR $c_l(\tau)$ fulfills “uncorrelated scattering” (US in $\tau$)

Channel IR vs. channel frequency response

**IDFT of transfer function after correction for linear phase shift**

**Amplitude transfer function**

**Phase transfer function (corrected for linear phase shift)**
Frequency-selective fading (cont’d)

Multipath intensity profile: average power output of channel as a function of delay

Characterization by parameters

- RMS delay spread
  \[ \tau_{rms} = \sqrt{\tau^2 - \bar{\tau}^2} \]
  second centralized moment of normalized multipath intensity profile

- mean and mean squared delay spread
  \[ \bar{\tau} = \frac{\int \tau S_c(\tau) d\tau}{\int S_c(\tau) d\tau} \quad \bar{\tau}^2 = \frac{\int \tau^2 S_c(\tau) d\tau}{\int S_c(\tau) d\tau} \]

- Coherence bandwidth
  \[ B_c \approx \frac{1}{\tau_{rms}} \]
Frequency-selective fading (cont’d)

Characterization of multipath intensity profile (simplified; suitable for indoor channels)

- Exponentially decaying part
- Line-of-sight (LOS) component
- Defined by channel parameters

Channel parameters:
- total power $P_0$
- K-factor (rel. strength of LOS)
- RMS delay spread (duration)

The WSSUS channel

- joint modeling of
  - time dispersion (= frequency selectivity)
  - and time variability (= Doppler spread)
- Define: ACF of time-variant CIR $c_l(\tau; t)$

$$E\{c_l^*(\tau_1; t)c_l(\tau_2; t + \Delta t)\} = \phi_c(\tau_1; \Delta t)\delta(\tau_1 - \tau_2)$$

assumes:
- time-variations are wide-sense stationary (WSS)
- attenuation and phase shifts are independent at $\tau_1$ and $\tau_2$: uncorrelated scattering (US)

- for $\Delta t = 0$: $\phi_c(\tau; \Delta t) = S_c(\tau)$ multipath intensity profile
- $\phi_c(\tau; \Delta t)$ ... lagged-time correlation function
The WSSUS channel (cont’d)

An equivalent representation of the t-var. CIR \( c_l(\tau; t) \):

- **Time-variant channel transfer function (TF)** \( C_l(f; t) \)

\[
c_l(\tau; t) \xrightarrow{\mathcal{F}_\tau} C_l(f; t) = \int_{-\infty}^{\infty} c_l(\tau; t) e^{-j2\pi f \tau} d\tau
\]

♦ from US property follows WSS in \( f \)-domain

- **equivalent characterization (ACF)**

\[
\phi_C(\Delta f; \Delta t) = \mathbb{E}\{C_l^*(f; t)C_l(f + \Delta f; t + \Delta t)\}
\]

spaced-frequency spaced-time correlation function (WSSWSS!)

The WSSUS channel (cont’d)

- **time- and frequency-selective transfer function**
The WSSUS channel (cont’d)

- **Equivalent representations**: time-variant system functions – Bello functions [Bello63]
  - $c_l(\tau; t)$ and $C_l(f; t)$ and **two more** Fourier transformed functions w.r.t. $t \leftrightarrow \nu$ and $f \leftrightarrow \tau$

- **Equivalent (2-nd order) characterizations**: correlation functions of Bello functions
  - $\phi_c(\tau; \Delta t)$ and $\phi_C(\Delta f; \Delta t)$ and **two more** Fourier transformed functions w.r.t. $\Delta t \leftrightarrow \nu$ and $\Delta f \leftrightarrow \tau$

Overview shown on next slide
The WSSUS channel (cont’d)

- Doppler-delay scattering function

![Doppler-delay scattering function graph](image)

[Paulraj; fig 2-9]

Channel as a space-time random field

- **Homogenous (HO) channel** is (locally) stationary in space
  - characterization:
    \[
    E\{c_l^*(\tau; t; d) c_l(\tau; t; d + \Delta d)\} = \phi_d(\tau; t; \Delta d)
    \]
  - agrees with discrete scattering model: each scatterer has discrete ToA \(\tau_i\) and AoA \(\theta_i\)

- space-angle transform: assume \(d\) lies on \(x\)-axis; parameterized by \(x\) (and dropping \(t\))
  \[
  c_l(\tau; x) = \int_{-\infty}^{\infty} c_l(\tau; \theta) e^{-j2\pi \sin(\theta) \frac{x}{\lambda}} d\theta
  \]

- we may define the **angle-delay scattering function** \(S_c(\tau; \theta)\)
Channel as a space-time random field (cont’d)

- Angle-delay scattering function

![Angle-delay scattering function]

*Figure 2.10: The angle-delay scattering function represents the average power in the angle-delay dimensions.*

[Paulraj; fig 2-10]

- **Characterization by parameters:**
  - **RMS angle spread:** \( \theta_{rms} = \sqrt{\theta^2 - \bar{\theta}^2} \)
    - second centralized moment of normalized angle power spectrum
  - **Mean and mean squared angle spread**
    - \( \bar{\theta} = \frac{\int \theta S_c(\theta) d\theta}{\int S_c(\theta) d\theta} \)
    - \( \bar{\theta}^2 = \frac{\int \theta^2 S_c(\theta) d\theta}{\int S_c(\theta) d\theta} \)
  - **Coherence distance** \( D_c \propto \frac{1}{\theta_{rms}} \)
3-3 Classification of Small-Scale Fading

- Compares system and channel parameters

<table>
<thead>
<tr>
<th>Classification</th>
<th>w.r.t. symbol period $T_s$</th>
<th>w.r.t. bandwidth $B_s \propto 1/T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dispersiveness</td>
<td>flat fading $T_s \gg \tau_{rms}$</td>
<td>$B_s \ll B_c$</td>
</tr>
<tr>
<td>frequency selective</td>
<td>$T_s \ll \tau_{rms}$</td>
<td>$B_s &gt; B_c$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>time variations</th>
<th>slow fading $T_s \ll T_c$</th>
<th>$B_s \gg \nu_{rms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast fading</td>
<td>$T_s &gt; T_c$</td>
<td>$B_s &lt; \nu_{rms}$</td>
</tr>
</tbody>
</table>

Classification example – GSM

- Key air-interface parameters:
  - Carrier frequency ... 900 MHz, 1.8 GHz
  - Bandwidth ... 200 kHz
  - Frame; slot length ... $\sim 4.6$ ms; $\sim 0.6$ ms

- Time dispersiveness
  - $\tau_{rms}$ (typical urban and suburban) ... 100–800 ns
  - corresponds to $B_c \approx 1.2–10$ MHz
  - flat fading

- Time variability
  - assume $v = 50$ m/s at $f_c = 1$ GHz $\Rightarrow \nu_{\text{max}} = 167$ Hz
  - corresponds to $T_c \approx 6$ ms
  - Time-invariant during slot
Classification example – WLAN

- **Key air-interface parameters:**
  - Carrier frequency ... 2.4; 5 GHz
  - Bandwidth ... 17 MHz (sampling f: \( f_s = 20 \) MHz)
  - OFDM symbol length ... 4 \( \mu s \)

- **Time dispersiveness**
  - \( \tau_{rms} \) (indoor) ... 10–300 ns
  - corresponds to \( B_c \approx 3–100 \text{ MHz} \)
  - frequency selective

- **Time variability**
  - assume \( v = 2 \text{ m/s} \) at \( f_c = 5 \text{ GHz} \) \( \rightarrow \nu_{\text{max}} = 33 \text{ Hz} \)
  - corresponds to \( T_c \approx 30 \text{ ms} \); several 1000 symbols
  - Time-invariant during packet