Advanced Signal Processing 1, SE

Distributed Source Coding

Johann Steiner

Outline

- Source Coding
- Distributed Source Coding
  - Slepian-Wolf Theorem
  - Slepian-Wolf Coding of Two Binary Sources
  - S-W Coding Example
  - SW-Coding of multiple sources with arbitrary correlation
- Wyner-Ziv Coding
  - WZ Coding, Binary Symmetric Case
  - WZ Coding ,The Quadratic Gaussian Case
  - Comparison of Different approaches of the Quadratic Gaussian Case
- Examples

Source Coding(1)

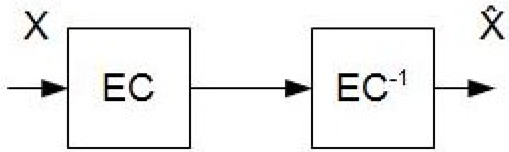
- Definition:

  source coding is the process of encoding information using fewer information-bearing units than an unencoded representation would use, through use of specific encoding schemes.

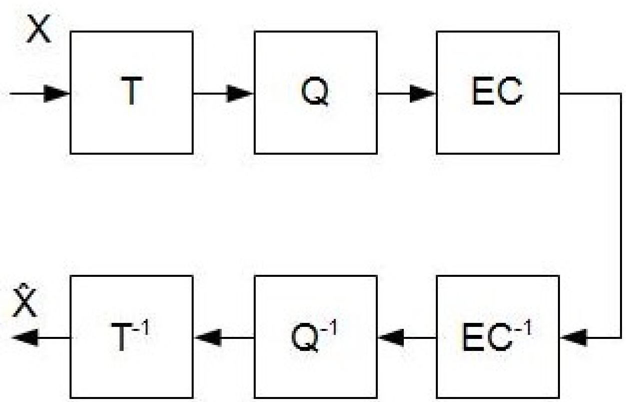
- Losless Source Coding
- Lossy Source Coding

Source Coding(2)

- Losless Source Coding



- Lossy Source Coding



Main Results in Source Coding

Entropy $H(X) = -E[\log p(X)] = -\sum_{x} p(x) \log p(x)$

Joint entropy $H(X,Y) = -E[\log p(X,Y)] = -\sum_{x,y} p(x,y) \log p(x,y)$

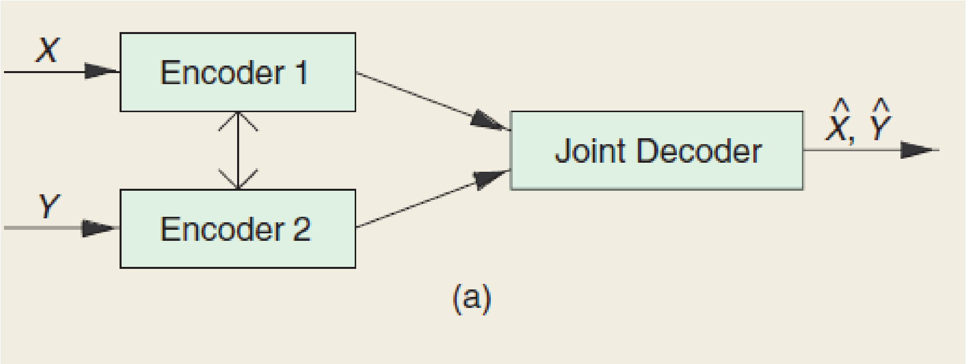
Conditional entropy $H(X|Y) = -E[\log p(X|Y)] = H(X,Y) - H(Y).$

Mutual information $I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$

Rate Distortion Function $R_X(D) = \min_{\substack{p(\hat{X}|X) \\ E[d(X,\hat{X})] \leq D}} I(X;\hat{X})$

Distributed Source Coding(1)

Encode a sequence of i.i.d drawings of a pair of correlated discrete random variables X and Y



from [1]

Use Encoders which work together (collaborated)

$R = H(X,Y)$ is sufficient for encoding X and Y together

Distributed Source Coding(2)

- Different parts of the source information may be available to separate encoding terminals that cannot cooperate.

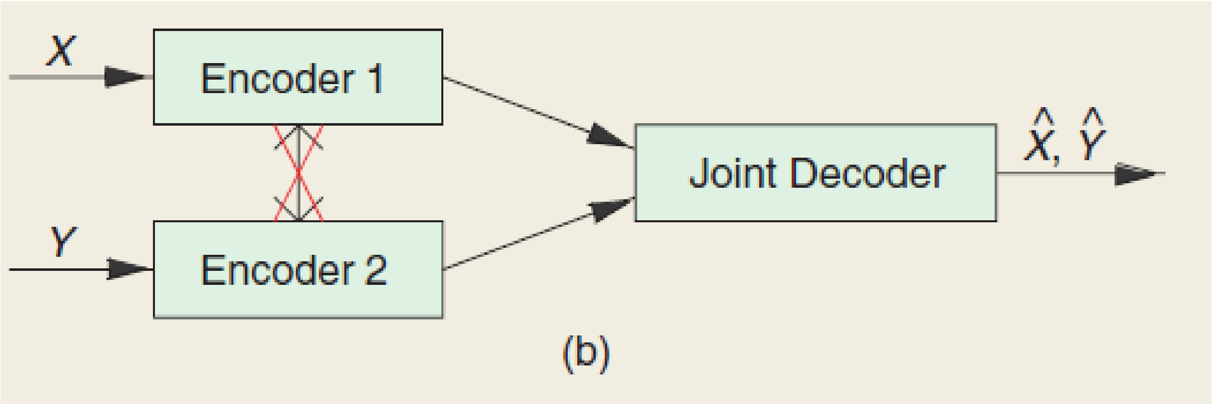
- Decoders may have access to additional side information about the source information;

  or they may only obtain a part of the description provided by the encoders.

From [6]

Distributed Source Coding(3)

Separately encoding of X and Y



from [1]

Simple way: Separate encode X and Y

$$R = H(X)+H(Y) > H(X,Y)$$

Slepian-Wolf Theorem(1)

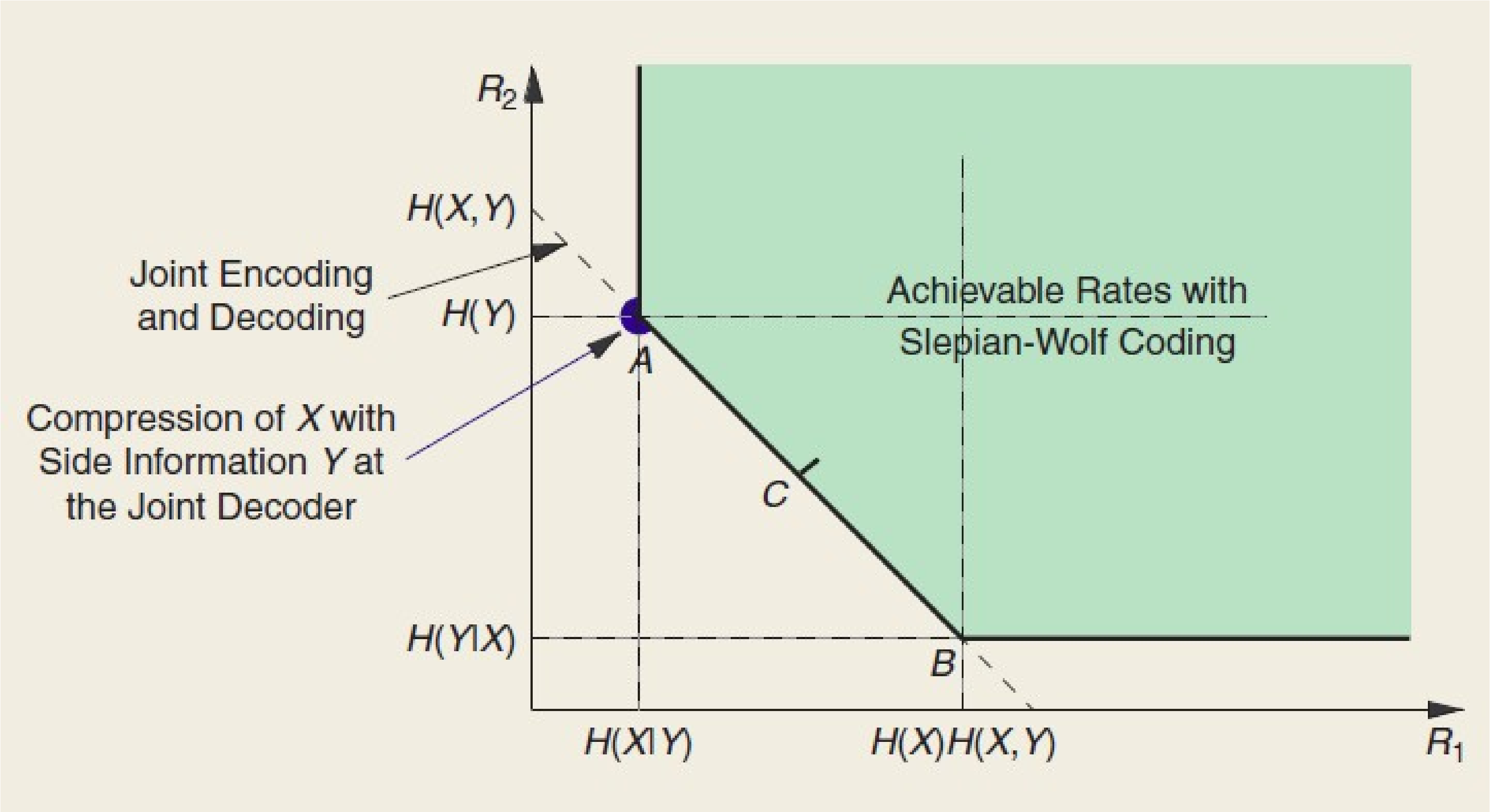
Given discrete memoryless sources X and Y, define $\mathfrak{R}$ as

$$\mathfrak{R} = \{(R1, R2) : R1 + R2 \geq H(X,Y), R1 \geq H(X|Y), R2 \geq H(Y|X)$$

Furthermore, let $\mathfrak{R}^0$ be the interior of $\mathfrak{R}$. Then $(R1, R2) \in \mathfrak{R}^0$ are achievable for the two terminal lossless source coding problem, and $(R1, R2) \notin \mathfrak{R}^0$ are not.

from [6]

Slepian-Wolf Theorem(2)



from [1]

Proof of the S.-W. Theorem(3)

Based on random binning:

partitioning the space of all possible outcomes of a random source into disjoint subsets called *bins*.

Example: (on Blackboard)

Slepian-Wolf Coding of Two Binary Sources

- Design a code to approach
  $R1 + R2 = H(X|Y) + H(Y) = H(X,Y)$ *point (A)*
- This is a problem of source coding of *X* with side information *Y* at the Decoder
- If this can be done, the point *(B) can also reached by swapping the role X and Y*
- All points between can be realized by time sharing

Slepian-Wolf Coding of Two Binary Sources

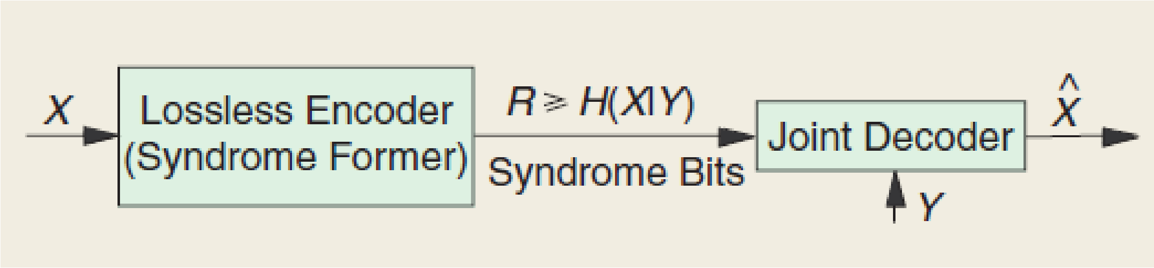
- Binary symmetric sources and Hamming distance measure with a linear *(n, k)* binary block code
- There are 2^{n-k} distinct syndromes, each indexing a bin of 2^k binary words of length n that preserve the Hamming distance properties of the original code.
- In compressing, a sequence of n input bits is mapped into its corresponding $(n - k)$ syndrome bits, achieving a compression ratio of $n : (n - k)$.

This approach, known as “Wyner’s scheme”

Slepian-Wolf Coding of Two Binary Sources

If the correlation between *X* and *Y* can be modeled by a binary channel

Wyner's syndrome concept can be extended to all binary linear codes (near-capacity channel codes)



from [1]

A more practical correlation model:

the binary symmetric model (BSC)

$$H(X\,|\,Y)=H(p)=-p\log2\,p-(1-p)\,\log2(1-p).$$

Slepian-Wolf Coding Example

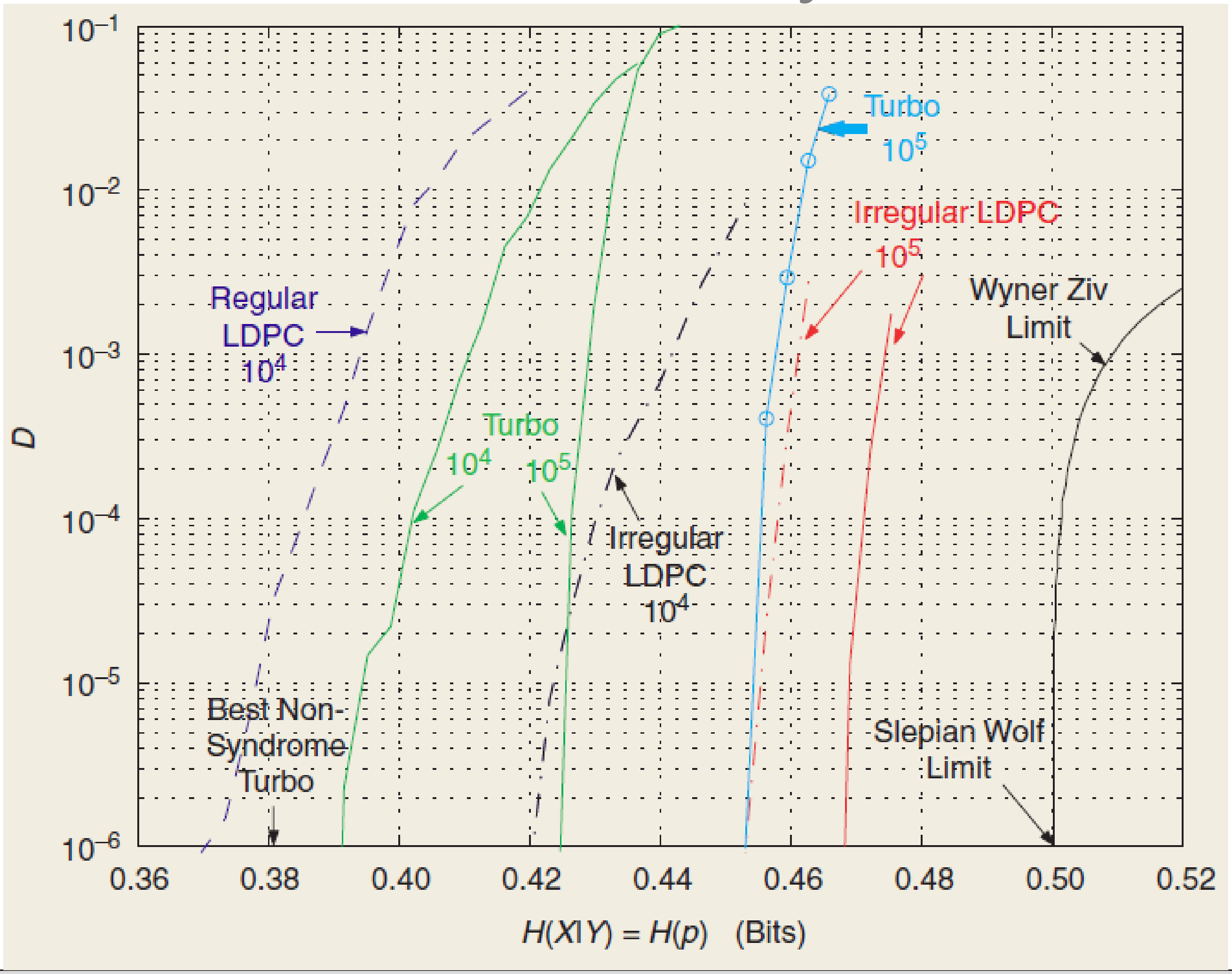
(On Blackboard)

SW-Coding of multiple sources with arbitrary correlation

"*No systematic approach to general practical Slepian-Wolf code design yet.* "[1]

In these more general problems, near-lossless compression of a source down to its entropy is a special case of Slepian-Wolf coding when there is no correlation either between the source and the side information (asymmetric setup) or between the different sources (symmetric setup).

SW-Coding of multiple sources with arbitrary correlation



- *Slepian-Wolf coding of binary source X with side information Y at the decoder*
- *based on turbo/LDPC codes.*
- *The rate for all codes is fixed at 1/2.*

from [1]

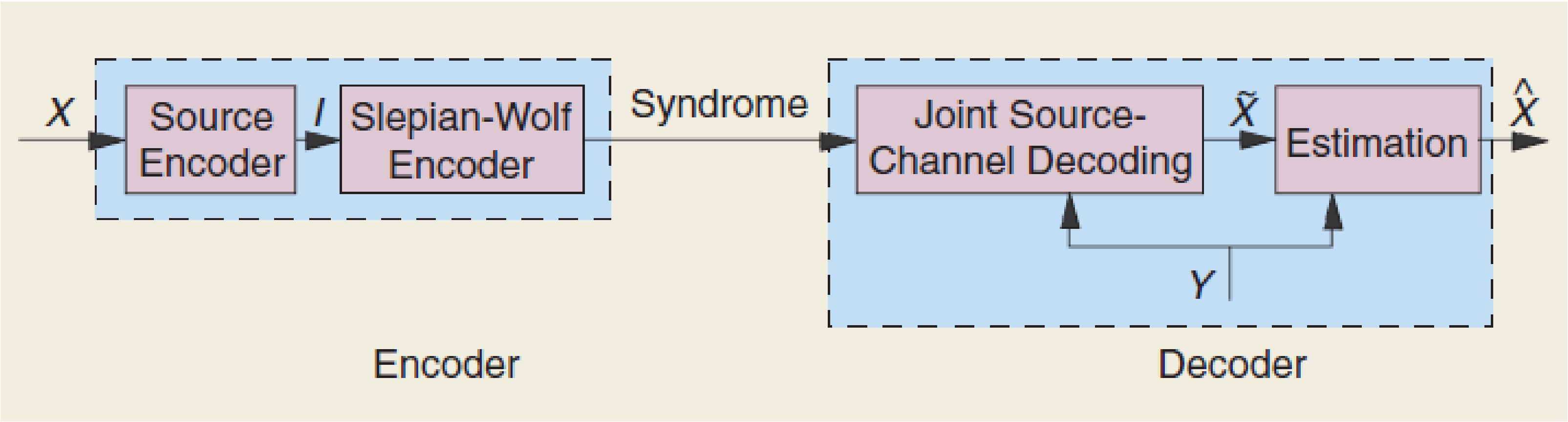
What's about Lossy Coding ?

- Slepian-Wolf coding combined with quantization provide a practical approach to lossy DSC problems.

→ Wyner-Ziv problem

- Similarly to the way quantization and entropy coding are combined in classic lossy source coding.

Wyner-Ziv Coding(1)



Block diagram of a generic Wyner-Ziv Coder

from [1]

Wyner-Ziv Coding(2)

The important thing about Wyner-Ziv coding is that it usually suffers rate loss when compared to lossy coding of X when the side information Y is available at both the encoder and the decoder.

One exception is when X and Y are jointly Gaussian with MSE measure. There is no rate loss with Wyner-Ziv coding in this case, which is of special interest in practice (e.g., sensor networks) because many image and video sources can be modeled as jointly Gaussian (after mean subtraction).

Wyner-Ziv Coding - rate-distortion function $R^*_{WZ}(D)$

- Binary Symmetric Case :
  - X,Y are binary symmetric sources
  - correlation model: BSC with crossover probability *p*
  - distortion measure : Hamming distance.

Performance limit $R_{X|Y}(D)$ of lossy coding of *X* given *Y* at both the encoder and the decoder

$$R_{X|Y}(D) = R_E(D) = \begin{cases} H(p) - H(D), & 0 \leq D \leq \min\{p, 1-p\}, \\ 0, & D > \min\{p, 1-p\}. \end{cases} \quad (3)$$

the Wyner-Ziv rate-distortion function in this case is

$$R^*_{WZ}(D) = l.c.e\{H(p * D) - H(D), (p, 0)\}, 0 \leq D \leq p, \quad (4)$$

from [1]

WZ Coding, BSC, Design Process

First, a good classic binary quantizer is selected, i.e., a quantizer that can minimize distortion *D* close to the distortion-rate function of a single Bernoulli(0.5) source at a given rate.

The second step is to design a Slepian-Wolf encoder matched to the quantizer codebook. The better the matching of the Slepian-Wolf code constraints (parity check equations) to the quantizer codebook, the better the performance of the decoder.

WZ Coding, Binary Symmetric Case

Use generalized Wyner's scheme using nested linear binary block codes

A linear (n, k_2) binary block code is again used to partition the space of all binary words of length n into 2^{n-k2} bins of 2^{k2} elements, each indexed by a unique syndrome value. Out of these 2^{n-k2} bins only 2^{k1-k2} $(k_1 \geq k_2)$ are used, and the elements of the remaining $2^{n-k2} - 2^{k1-k2}$ sets are "quantized" to the closest, in Hamming distance sense, binary word of the allowable $2^{k1-k2} \times 2^{k2} = 2^{k1}$ ones.

This "quantization" can be viewed as a (n, k_1) binary block source code.

WZ Coding, Binary Symmetric Case

Then the linear (n, k_2) binary block code can be considered

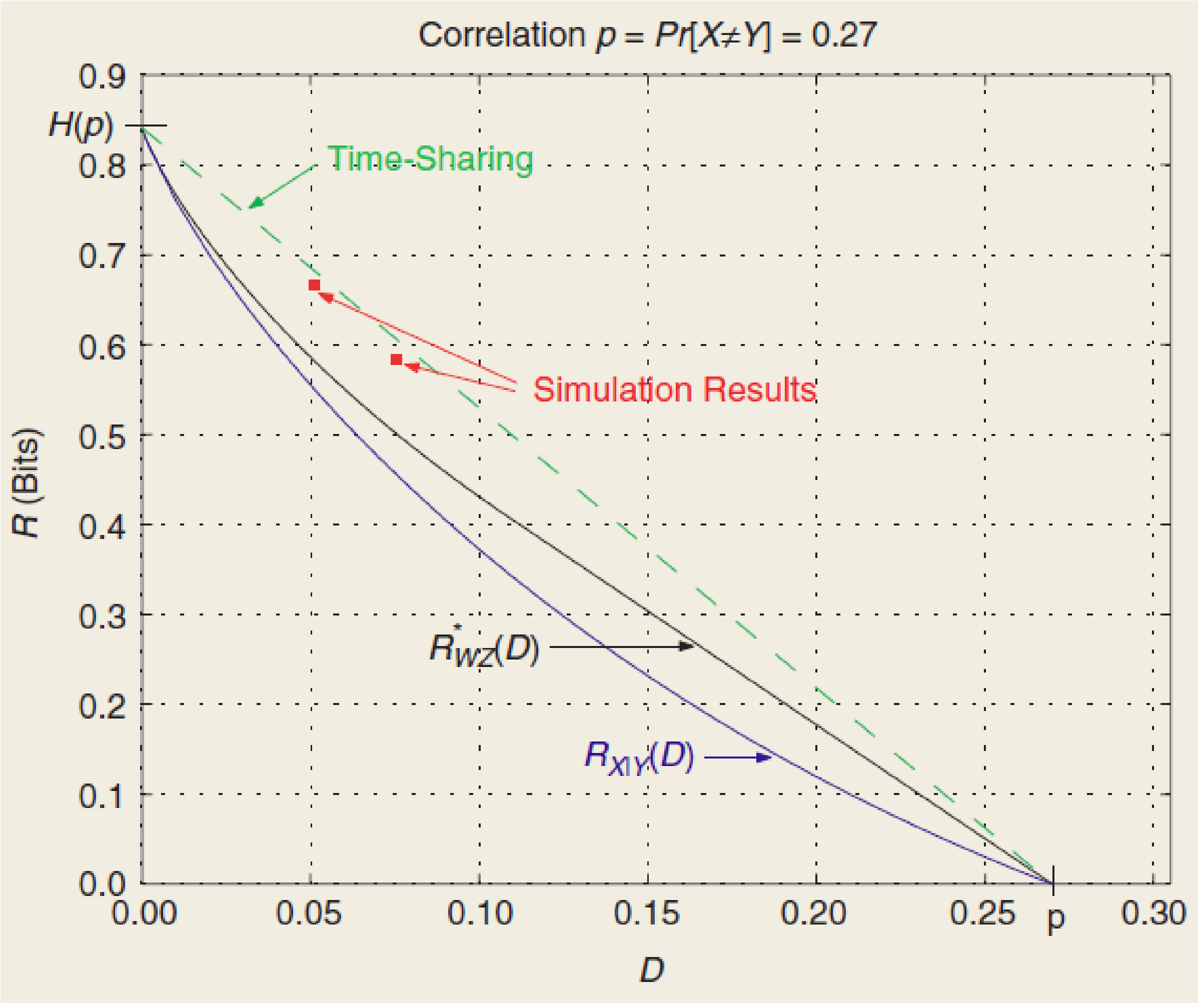
to be a coarse channel code nested inside the (n, k_1) fine source code.

To come close to the Wyner-Ziv limit, both codes in the above nested scheme should be good, i.e., a good fine source code is needed with a good coarse channel sub code.

WZ Coding, Binary Symmetric Case

The binary symmetric Wyner-Ziv problem does not seem to be practical, but due to its simplicity, it provides useful insight into the interaction between source and channel coding.

WZ Coding, Binary Symmetric Case



Simulated performance of the nested scheme for binary Wyner-Ziv coding for correlation $p = 0.27$.

from [1]

WZ Coding ,The Quadratic Gaussian Case

X and Y are jointly Gaussian with MSE measure.

→ There is no rate loss with Wyner-Ziv coding in this case, because many sources can be modeled as jointly Gaussian (after mean subtraction).

WZ Coding ,The Quadratic Gaussian Case

covariance matrix of X and Y (with $|\rho| < 1$) :

$$\Lambda = \begin{bmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix}$$

$$R^*_{WZ}(D) = R_{X|Y}(D) = \frac{1}{2}\log^+\left[\frac{\sigma_X^2(1-\rho^2)}{D}\right],$$

$\log^+ x = \max\{\log_2 x, 0\}$

from [1]

There is no rate loss with
Wyner-Ziv coding in this quadratic Gaussian case.

WZ Coding ,The Quadratic Gaussian Case

Consider lattice codes and trellis-based codes that have been used for both source and channel coding in the past and focus on finding good nesting codes among them.

Using simplest one-dimensional (1-D) nested lattice/scalar quantizer with N = 4 bins

(stepsize q, and disctance d_{min} = N q)

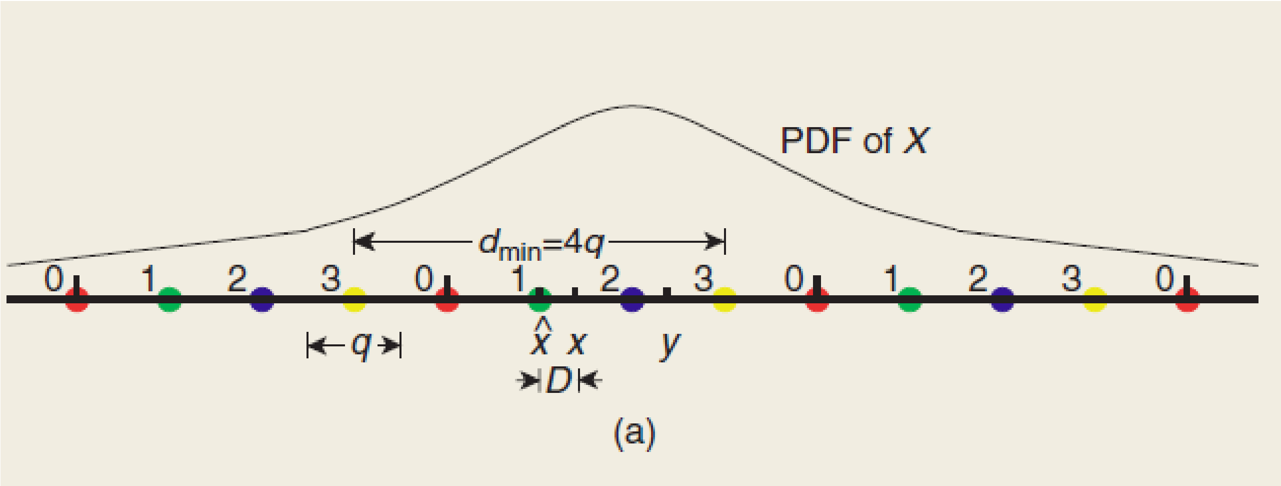
WZ Coding ,The Quadratic Gaussian Case

The distortion consists of two parts:

“good” distortion introduced from quantization by the source code

“bad” distortion from decoding error of the channel code.

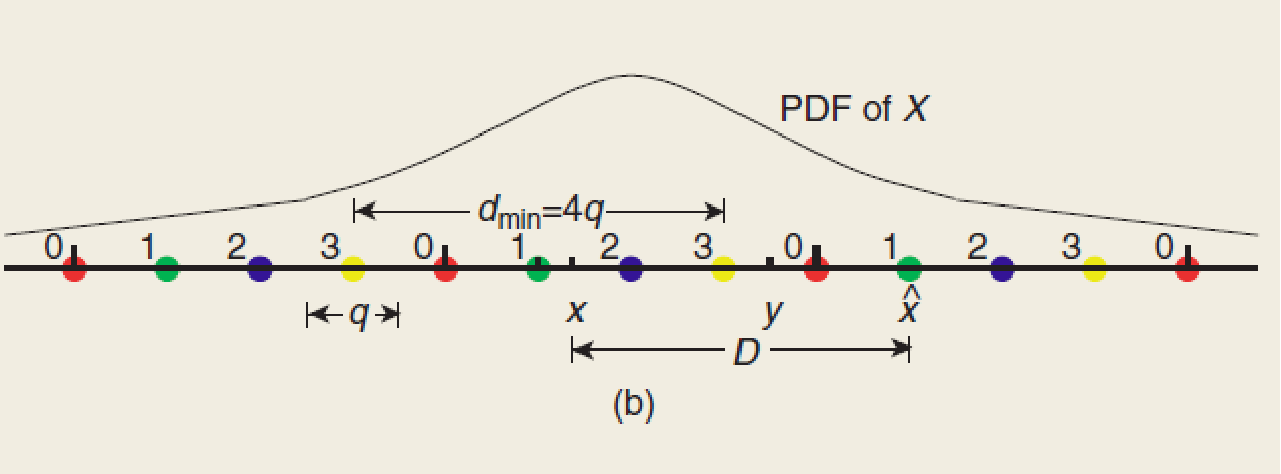
WZ Coding ,The Quadratic Gaussian Case



from [1]

1-D nested lattice/uniform quantizer with four bins for the quadratic Gaussian Wyner-Ziv problem, where Y is the side information only available at the decoder ("Good" distortion only)

WZ Coding ,The Quadratic Gaussian Case

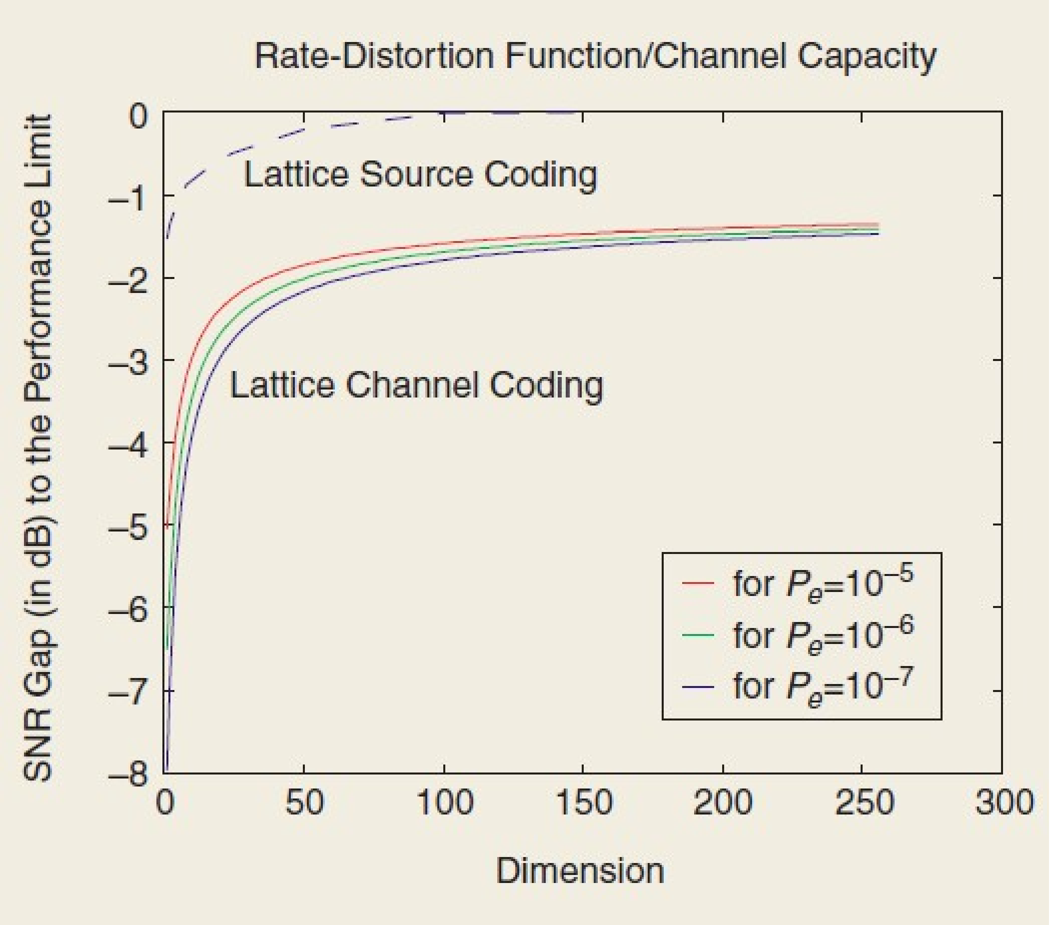


from [1]

1-D nested lattice/uniform quantizer with four bins for the quadratic
Gaussian Wyner-Ziv problem, where Y is the side information only available at the decoder (“Good” and “bad” distortion)

Lower Bounds in terms of performance Gap

- With nested scalar quantization, the fine source code (scalar quantization) leaves unexploited the maximum granular gain of only 1.53 dB
- coarse channel code (scalar coset code) suffers more than 6.5 dB loss with respect to the capacity (with *Pe* = 10−6*)*.
- lattice channel code at dimension 250 still performs more than 1 dB away from the capacity.
- 6-dB rule, i.e., that every 6 dB correspond
- to 1 b, which is approximately true for both source and channel coding.



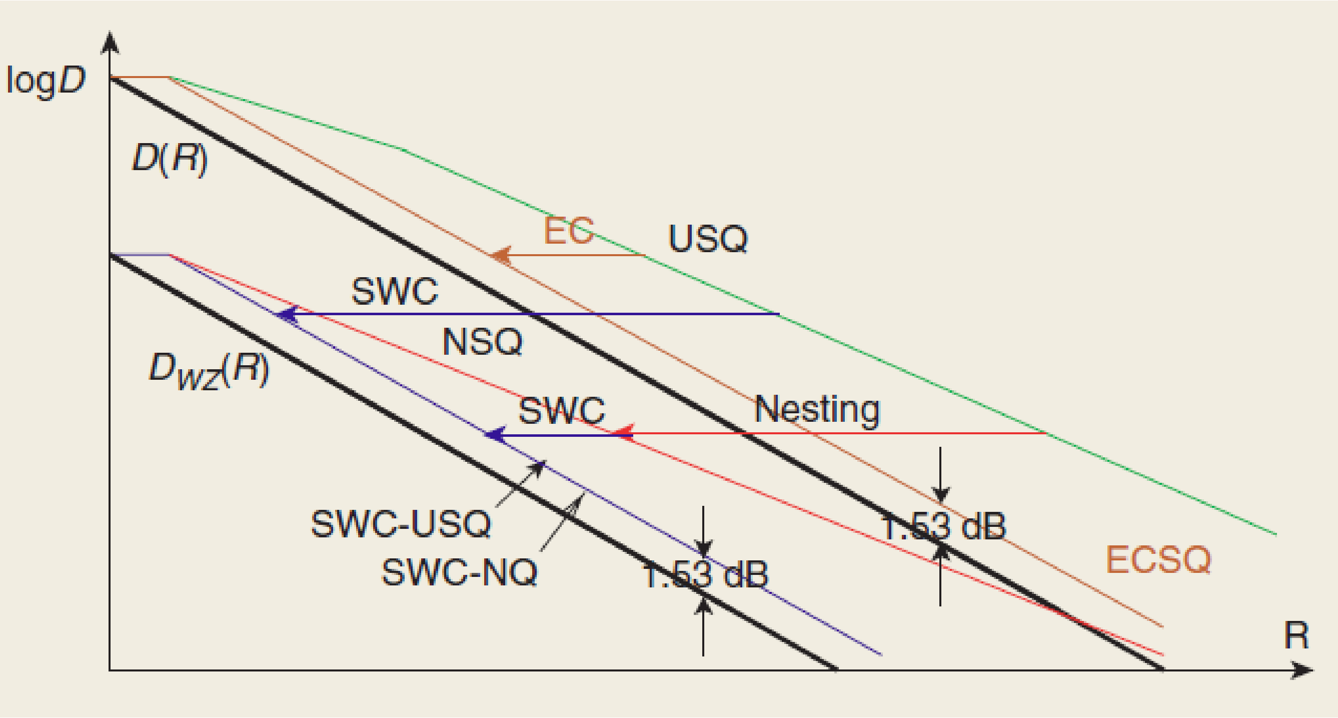
from [1]

Comparison of Different approaches of the Quadratic Gaussian Case

Based on scalar quantization:

- Classical scalar uniform Quantization (USQ)
- Nested Scalar Quantization (NSQ)
- Slepian-Wolf coded Nested Quantization (SWC-NQ)
- Slepian-Wolf coded uniform scalar quantization (SWC-USC)

Comparison of Different approaches of the Quadratic Gaussian Case



from [1]

High-rate classic source coding versus high-rate Wyner-Ziv coding

Comparison of Different approaches of the Quadratic Gaussian Case

Table 1. High-rate classic source coding versus high-rate Wyner-Ziv coding.

Classic Source Coding		Wyner-Ziv Coding	
Coding Scheme	Gap to $D_X(R)$	Coding Scheme	Gap to $D^*_{WZ}(R)$
ECSQ [24]	1.53 dB	SWC-NSQ [30]	1.53 dB
ECLQ (2-D) [28]	1.36 dB	SWC-NQ (2-D) [30]	1.36 dB
ECTCQ [24]	0.2 dB	SWC-TCQ [32]	0.2 dB

Values for the before shown figure, taken from [1]

Examples

- Sensor Networks
- Distributed Video Coding
- Robust Video Transmission
- Securing Biometric Data
- Distributed Compression in Microphone Arrays

from [6]

Bibiliography

[1] Z. Xiong, A.D. Liveris, S. Cheng: "Distributed Source Coding for Sensor Networks," IEEE Signal Processing Magazine, Sept. 2004.

[2] B. Girod et al: "Distributed Video Coding," Proc. IEEE, Vol.93, No.1, Jan. 2005.

[3] D. Schonberg, K. Ramchandran, S.S. Pradhan: "Distributed code constructions for the entire Slepian-Wolf rate region for arbitrarily correlated sources," Proc. Data Compression Conference, 2004.

[4] D.Slepian and J.K. Wolf:“Noiseless coding of correlated information sources,“ IEEE Transaction on information theory vol. 19

[5] C. Feldbauer, W.B.Kleijn, “Practical Source Coding for Sensor Networks”, IEEE Transaction on communication, Vol. 99, Jan2009

[6] M.Gastpar, P.L. Dragotti, “Distributed source coding: theory, algorithms and applications”, Academic Press, 2009