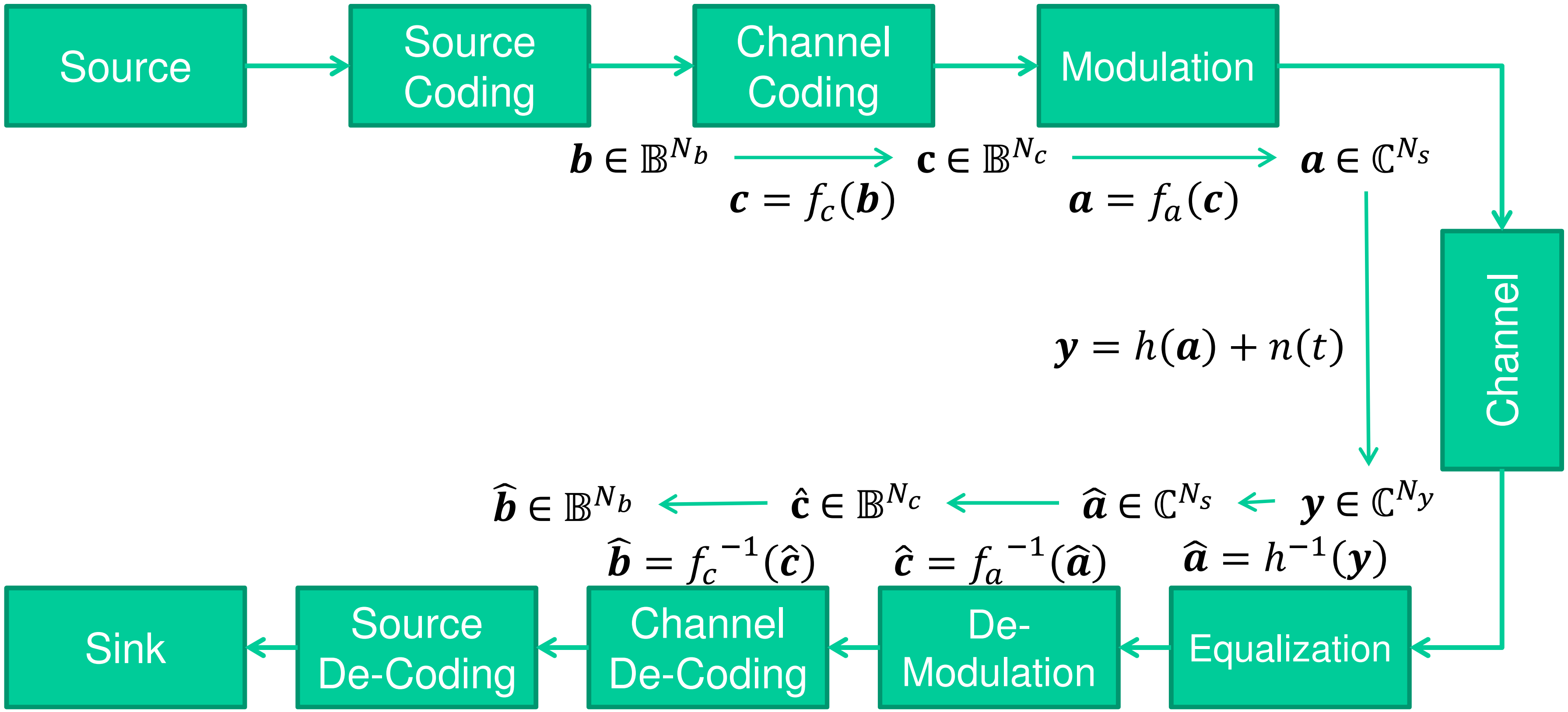
Iterative Receiver

Outline

- Digital Communication
- From Receiver Design to Factor Graph
- Channel-Decoding
- De-Modulation
- Equalization
- Synchronization
- All-In-One
- Summary

Classical Digital Communication



Baseband Equivalent Model

Receiver Estimation Process

Aim of a receiver: Estimation of the transmitted data (bits) in an "optimal" sense

- Sequence/Frame Detection

$$\hat{\boldsymbol{b}} = \arg\max_{\boldsymbol{b}\in\mathbb{B}^{N_b}} \{p(\boldsymbol{B} = \boldsymbol{b}|\boldsymbol{Y} = \boldsymbol{y}, \mathcal{M})\}$$

  - Maximizing the a-posteriori probability of a data frame is equivalent to minimizing the frame error rate (FER)

- Bit-by-Bit Detection

$$\hat{b}_k = \arg\max_{b_k\in\mathbb{B}} \{p(B_k = b_k|\boldsymbol{Y} = \boldsymbol{y}, \mathcal{M})\}$$

  - Maximizing the a-posteriori probability of a single data bit is equivalent to minimizing the bit error rate (BER)

- →MAP Estimation

Factor Graphs in Digital Communication

- The needed A-Posteriori $p(B_k = b_k|\boldsymbol{Y} = \boldsymbol{y}, \mathcal{M})$ is given as:

$$p(B_k = b_k|\boldsymbol{Y} = \boldsymbol{y}, \mathcal{M}) = \frac{p(B_k = b_k, \boldsymbol{Y} = \boldsymbol{y}|\mathcal{M})}{\sum_b p(B_k = b_k, \boldsymbol{Y} = \boldsymbol{y}|\mathcal{M})}$$

- Factorization of the joint Probability $p(\boldsymbol{B}, \boldsymbol{Y} = \boldsymbol{y}|\mathcal{M})$:

$$p(\boldsymbol{B}, \boldsymbol{Y} = \boldsymbol{y}|\mathcal{M}) = p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{B}, \mathcal{M})p(\boldsymbol{B}|\mathcal{M})$$

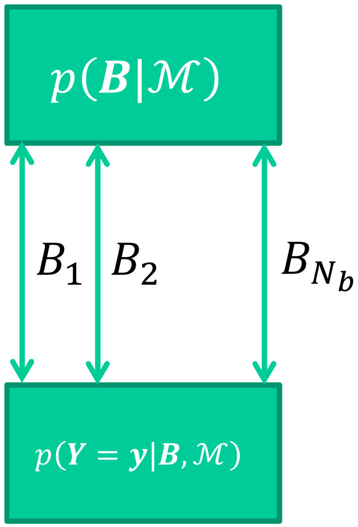
- A-Priori knowledge of the transmitted Bits:

$$p(\boldsymbol{B} = \boldsymbol{b}|\mathcal{M}) = \prod_{k=1}^{N_b} p(B_k = b_k|\mathcal{M})$$

- How does the Likelihood $p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{B}, \mathcal{M})$ look like:

$$p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{B}, \mathcal{M}) = ???$$

→ **"Opening" the Factor!**



Channel De-Coding (1)

- „Opening“ the Factor $p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{B},\mathcal{M})$:

$$p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{B},\mathcal{M}) = \sum_{\boldsymbol{c}} p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{C}=\boldsymbol{c}|\boldsymbol{B},\mathcal{M})$$

- Factorization of the joint Probability $p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{C}|\boldsymbol{B},\mathcal{M})$:

$$\begin{aligned} p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{C}|\boldsymbol{B},\mathcal{M}) &= p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\boldsymbol{B},\mathcal{M})p(\boldsymbol{C}|\boldsymbol{B},\mathcal{M}) \\ &= p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\mathcal{M})p(\boldsymbol{C}|\boldsymbol{B},\mathcal{M}) \end{aligned}$$

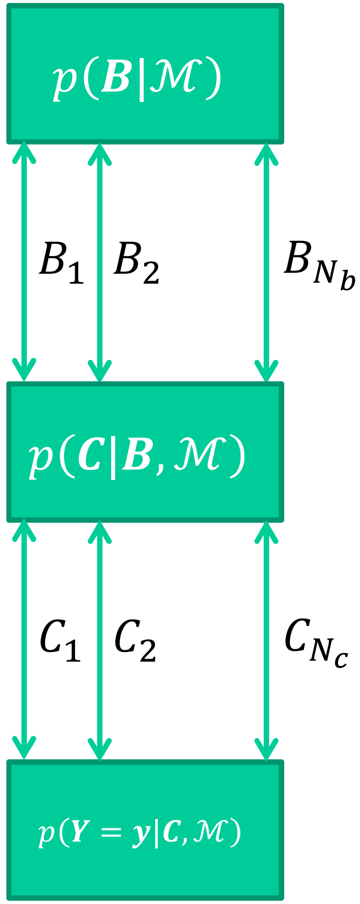
- The factor $p(\boldsymbol{C}|\boldsymbol{B},\mathcal{M})$ describes the coding scheme:

$$p(\boldsymbol{C}=\boldsymbol{c}|\boldsymbol{B}=\boldsymbol{b},\mathcal{M}) = \mathbb{I}(\boldsymbol{c},f_c(\boldsymbol{b}))$$

$$\mathbb{I}\left(\boldsymbol{c},f_c(\boldsymbol{b})\right) = \begin{cases} 1, \boldsymbol{c} = f_c(\boldsymbol{b}) \\ 0, \boldsymbol{c} \neq f_c(\boldsymbol{b}) \end{cases}$$

- Again: How does the Likelihood $p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\mathcal{M})$ look like:

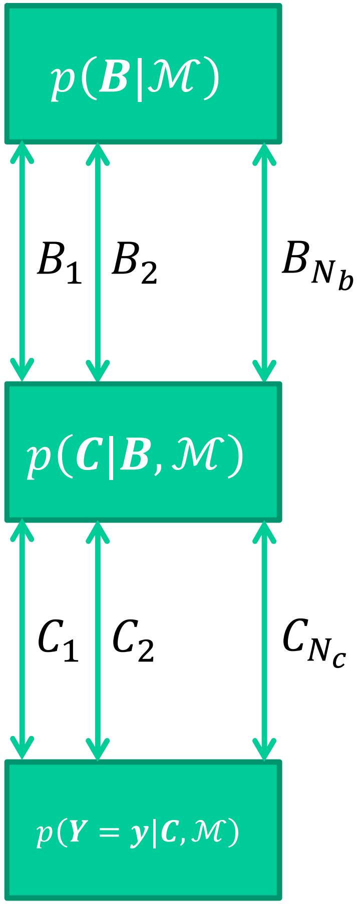
$$p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\mathcal{M}) = ???$$



Channel De-Coding (2)

Example (1/4 Repetition Code):

- Known:
  - Data Prior:
    $$p(B=0)=0.5$$
  - Channel Likelihood:
    $$p(Y=0|C=0)=0.5$$
    $$p(Y=0|C=1)=0.1$$
- Given Observation:
  $$\boldsymbol{y}=[0,0,1,1]$$
- Find: b

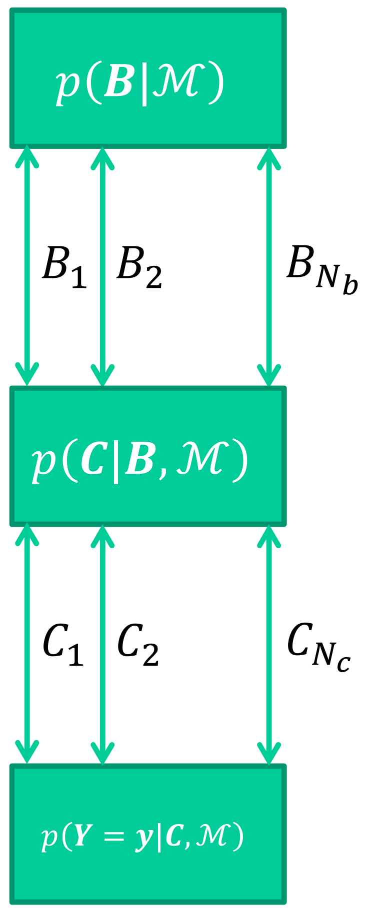


Channel De-Coding (3)

Example (1/4 Repetition Code), ctd:

$$p(B, \boldsymbol{Y} = \boldsymbol{y}) = p(\boldsymbol{Y} = \boldsymbol{y} | B) p(B)$$
$$= \sum_{\boldsymbol{c}} p(\boldsymbol{Y} = \boldsymbol{y}, \boldsymbol{C} = \boldsymbol{c} | B) p(B)$$

$$p(\boldsymbol{Y} = \boldsymbol{y}, \boldsymbol{C} | B)$$
$$= p(\boldsymbol{Y} = \boldsymbol{y} | \boldsymbol{C}, B) p(\boldsymbol{C} | B)$$
$$= \prod_{k} p(Y_k = y_k | C_k) \mathbb{I}(C_k, B)$$



Channel De-Coding (4)

Example (1/4 Repetition Code), ctd:

with $\boldsymbol{y} = [0,0,1,1]$

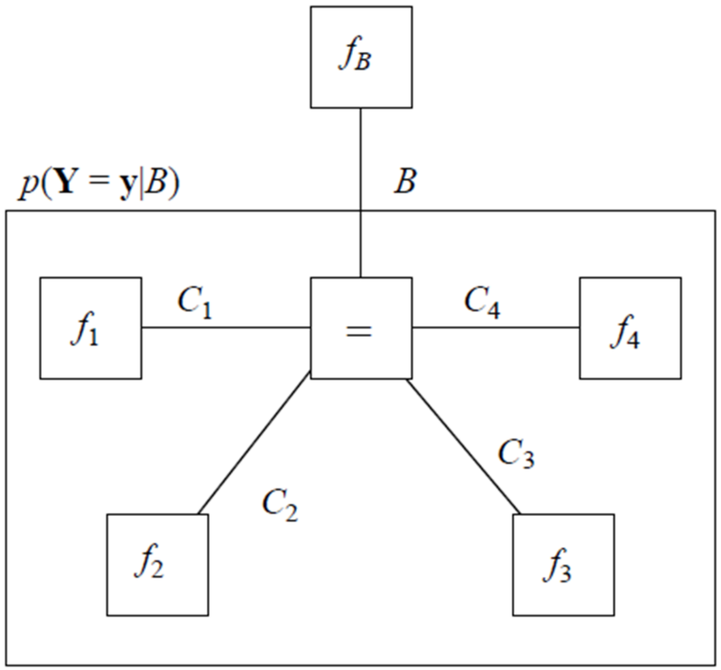
$$\mu_{f_B \to B}(b) = \begin{cases} 0.5; & b = 1 \\ 0.5; & b = 0 \end{cases}$$

for k =1,2

$$\mu_{f_k \to B}(b) = \begin{cases} 0.1; & b = 1 \\ 0.5; & b = 0 \end{cases}$$

for k =3,4

$$\mu_{f_k \to B}(b) = \begin{cases} 0.9; & b = 1 \\ 0.5; & b = 0 \end{cases}$$



Channel De-Coding (5)

Example (1/4 Repetition Code), ctd:

$$\mu_{=\to B}(B=b) = \sum_{\boldsymbol{c}} \mathbb{I}(b, c_1, c_2, c_3, c_4) \prod_k \mu_{f_k \to C_k}(c_k) = \prod_k \mu_{f_k \to C_k}(b)$$

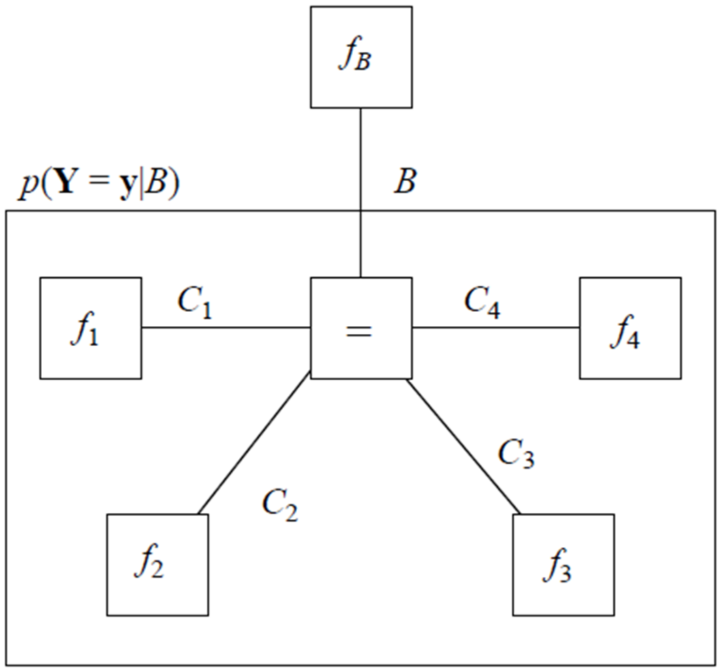
$$\mu_{=\to B}(B=0) = (0.5)^4 = 0.0625$$
$$\mu_{=\to B}(B=1) = (0.9)^2(0.1)^2 = 0.0081$$

$$g_B(B=b) = \mu_{f_B \to B}(b)\mu_{=\to B}(b) = p(B=b, \boldsymbol{Y}=\boldsymbol{y}):$$

$$g_B(B=1) = 0.00405$$
$$g_B(B=0) = 0.03125$$

$$p(B=b|\boldsymbol{Y}=\boldsymbol{y}) = \frac{p(B=b, \boldsymbol{Y}=\boldsymbol{y})}{\sum_{\boldsymbol{b}} p(B=b, \boldsymbol{Y}=\boldsymbol{y})}:$$

$$p(B=1|\boldsymbol{Y}=\boldsymbol{y}) = \mathbf{0.11}$$
$$p(B=0|\boldsymbol{Y}=\boldsymbol{y}) = \mathbf{0.89}$$



De-Modulation (1)

- „Opening“ the Factor $p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\mathcal{M})$:

$$p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{C},\mathcal{M}) = \sum_{\boldsymbol{c}} p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{A}=\boldsymbol{a}|\boldsymbol{C},\mathcal{M})$$

- Factorization of the joint Probability $p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{A}|\boldsymbol{C},\mathcal{M})$:

$$\begin{aligned} p(\boldsymbol{Y}=\boldsymbol{y},\boldsymbol{A}|\boldsymbol{C},\mathcal{M}) &= p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{A},\boldsymbol{C},\mathcal{M})p(\boldsymbol{A}|\boldsymbol{C},\mathcal{M}) \\ &= p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{A},\mathcal{M})p(\boldsymbol{A}|\boldsymbol{C},\mathcal{M}) \end{aligned}$$

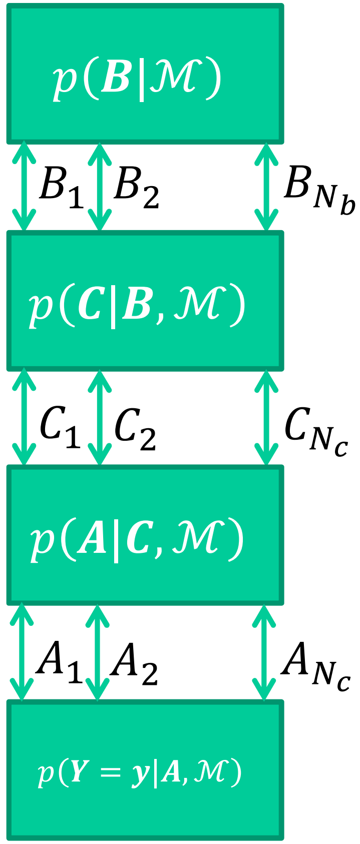
- The function $p(\boldsymbol{A}|\boldsymbol{C},\mathcal{M})$ describes the modulation scheme:

$$p(\boldsymbol{A}=\boldsymbol{a}|\boldsymbol{C}=\boldsymbol{c},\mathcal{M}) = \mathbb{I}(\boldsymbol{a},f_a(\boldsymbol{c}))$$

$$\mathbb{I}\left(\boldsymbol{a},f_a(\boldsymbol{c})\right) = \begin{cases} 1, \boldsymbol{a} = f_a(\boldsymbol{c}) \\ 0, \boldsymbol{a} \neq f_a(\boldsymbol{c}) \end{cases}$$

- And Again: How does the Likelihood $p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{A},\mathcal{M})$ look like:

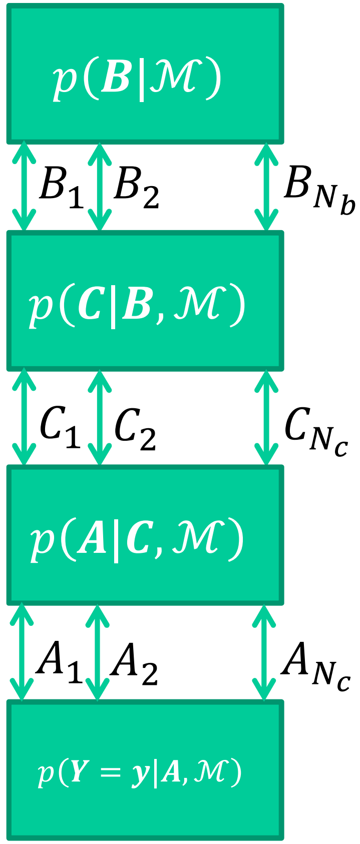
$$p(\boldsymbol{Y}=\boldsymbol{y}|\boldsymbol{A},\mathcal{M}) = ???$$



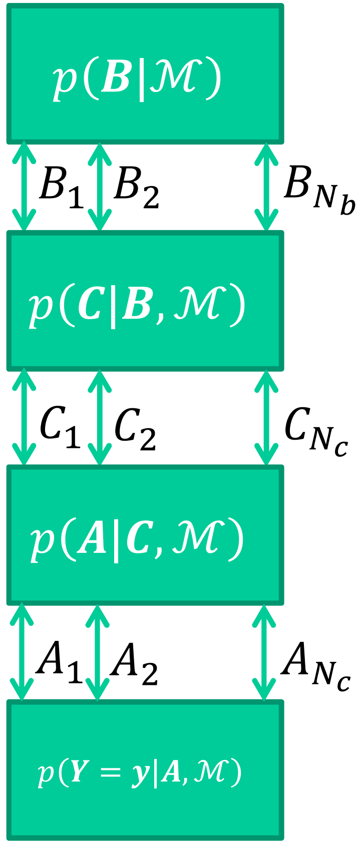
Equalization (1)

- The likelihood function $p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{A}, \mathcal{M})$ gives the likelihood that the observed signal $\boldsymbol{y}$ represents a given modulation symbol $\boldsymbol{a}$.

- Equalization estimate the channel which has an influence on the transmitted symbol



Equalization (2)



Example (BPSK):

BPSK symbol $a \in \{-1,+1\}$ with the mapping function

$$a_n = f_c(c_n) = 2c_n - 1$$

Considering AWGN channel

$$\boldsymbol{y} = \boldsymbol{a} + \boldsymbol{n}$$

with $\boldsymbol{n}$ is a RV with $\mathcal{N}(0,\sigma^2)$:

$$p(Y_k = y_k|A_k,\mathcal{M}) \propto exp\left(-\frac{(y_k - A_k)^2}{2\sigma^2}\right)$$

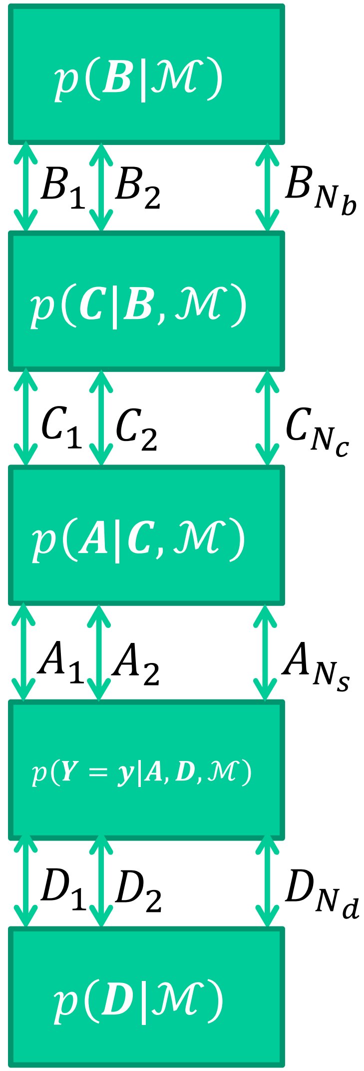
Synchronization (1)

- Until now the (deterministic) channel h was considered to be known

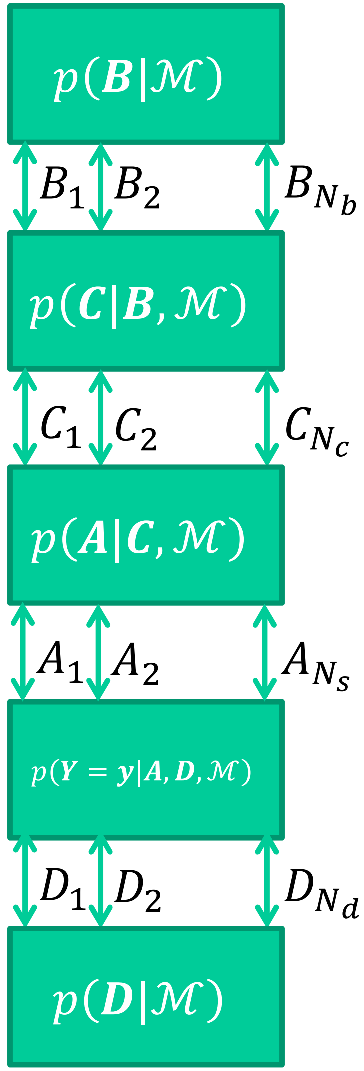
- Synchronization introduces an statistical channel approach using the channel parameters $\boldsymbol{D}$

- Therefore the factor $p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{A}, \mathcal{M})$ is expanded for due to the statistical channel model

$$p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{A}, \mathcal{M}) = p(\boldsymbol{Y} = \boldsymbol{y}|\boldsymbol{A}, \boldsymbol{D}, \mathcal{M})p(\boldsymbol{D}|\mathcal{M})$$



Synchronization (2)



Example (Multipath Channel):

- The multipath channel is typically denoted as

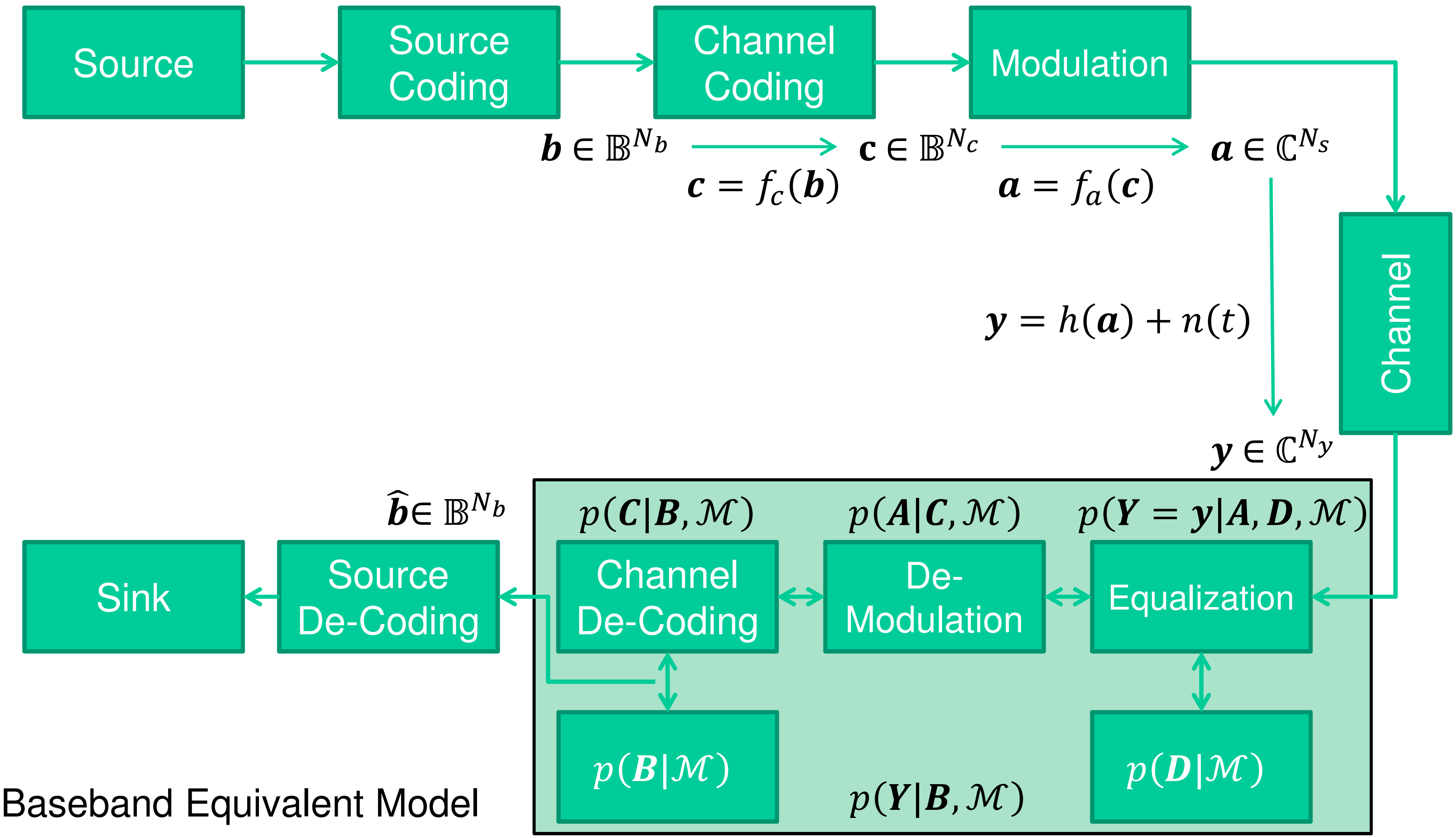
$$h(t) = \sum_{k=0}^{K} \alpha_k \delta(t - \tau_k)$$

- Therefore:

$$\boldsymbol{D} \epsilon \{\boldsymbol{\alpha}, \boldsymbol{\tau}\}^{K+1}$$

with $\boldsymbol{\alpha}, \boldsymbol{\tau}$ are RV with specific distributions

Iterative Receiver Design



Baseband Equivalent Model

Summary

- A classical receiver design can be fully substituted by an iterative approach
- Factor Graphs provide a very good framework to design the different stages e.g.
  - Source De-Coding
  - De-Modulation
  - Equalization
  - Synchronization
- The full receiver factor graph has loops (also between different blocks)
- Iterative Receiver Design gives the ability to distribute estimation information „backward“