Sensing Reality and Communicating Bits: A Dangerous Liaison[1]

Is digital communication sufficient for sensor networks?

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Introduction (1)

- Goal of Sensor Networks: Capture spatiotemporal variations of the underlying signals
- When is it sufficient to work with discrete-time, discrete-space and discrete-amplitude representations?
- Is there a spatiotemporal sampling theorem for typical datasets in sensor netwoks?
- The answers are based on the physics of the underlying process

Introduction (2)

- Signal processing and communications have become separate topics over the past several decades
- Sensor Networks: Address signal processing and communication jointly
  - Impossible to ship all data to a central location before processing it
  - Process the data (at least partly) in a distributed fashion at the sensors
  - Reduce communication needs and power supplys
- Either we go to the digital domain and apply discretization of data trough quantization and source compression

  → source channel separation
- Or keep data in analog form

  → joint source-channel coding

Source channel separation (1)

- „To separate or not to separate“
- Can an optimal coding strategy be implemented by first compressing the source(s) into bit streams and communicate those via error-correcting codes?
- The answer crucially depends on the interplay between the source structure, the observation process and the communication infrastructure.
- Later we will illustrate this with example scenarios

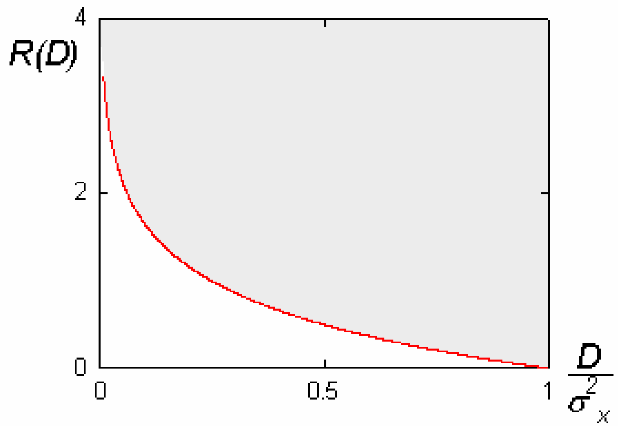
Source channel separation (2)

- Source channel separation theorem (Shannon, 1948)
- If the source coding rate of a given source is strictly below the channel capacity, then the source can be reliable transmitted trough the channel by appropriate encoding and decoding operations
- Reliable transmission is possible by separate source channel coding or not possible at all
- Expressed in terms of rate-distortion R(D) and channel capacity C
- With the separation theorem a distortion D is attainable if

  $$R(D) < C$$

  and is not attainable if

  $$R(D) > C$$

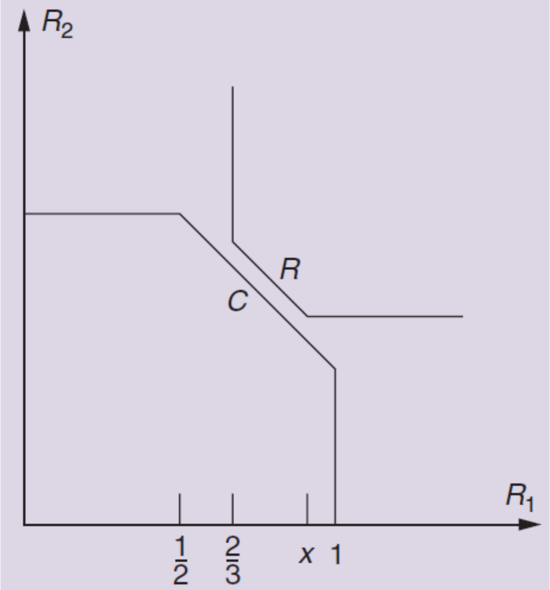


Source channel separation (3)

- Multidimensional case for sensor networks
- MAC scenario (Multiple access channel)
  - Two terminals transmit with power P, each on the same frequency band to a single base station
- Analog for the point to point case we now consider rate vectors (R_1 … R_L) in bits/symbol
- Rate Region R and capacity region C
- Distortion D is is attainable if the intersection between R and C is not empty

  $R \cap C \neq 0$
- If there is no intersection region there is no digital solution for the problem, but there may exist an analog architecture wich acheives the desired distortion

  → **Joint source-channel coding**

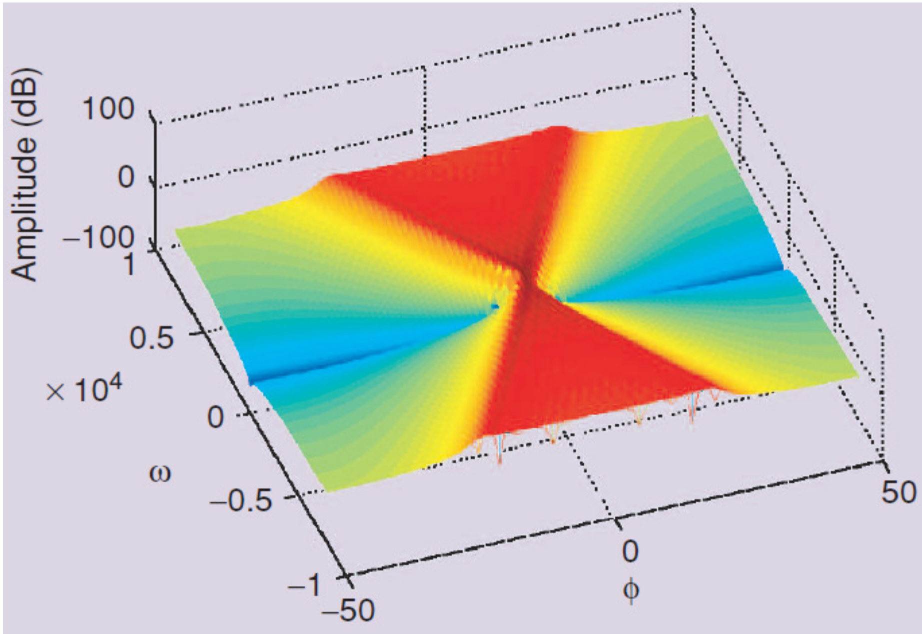


Discrete Space

- *„Discretization in time is an engineers choice while discretization in space is a physical necessity“* [1]
- Spatial low-pass filtering before sampling is impossible
  → Most sensor data is aliased with respect to spatial frequency
- Methods similar to *Array signal processing* but „the array“ is irregular and two dimensional (random sensor placement on a plane)
- Examples
  - Plenacoustic function
  - Plenoptic function (distributed camera systems)
  - Distribution of temperature (heat equation)

Discrete Space : The plenacoustic function[1,3]

- The sound field in a room is the solution of the Second order Wave equation
- The plenacoustic function (Green Function) is a Kernel of the wave equation
- It‘s Fourier fransorm it‘s bandlimited in spatial frequency
- Typical boq-tie shape of the spectrum



Φ…spatial frequency [1/m]

ω...temporal frequency [1/s]

c…speed of sound [m/s]

d…distance between mic. [m]

$\Phi \leq \omega / c$

$\Phi_{MAX} = \omega_0 / c$

$d = c / \omega_0$

Digital Architectures

- A digital architecture is a two stage scenario
  - The source code is designed with only the capacity region
  - The channel code is designed without any knowledge about the source at all
- Source coding/compressing tools
  - Vector quantizer followed by entropy coder
  - KLT (Karhunen Loeve transform)
- On the Cannel: Error correcting code that avoids (block) errors
- Multiple Correlated sources in a Sensor Network Problem

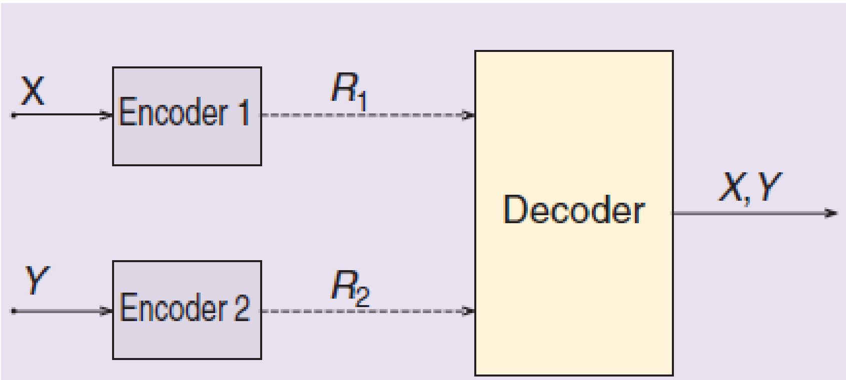
  → distributed source coding

  (Slepian and Wolf Theorem)
- Three scenarios
  - Expanding Sensor Network
  - Refining Sensor Network

    → comarsion to analog important
  - Camera Sensor Network

Distributed source coding (1)

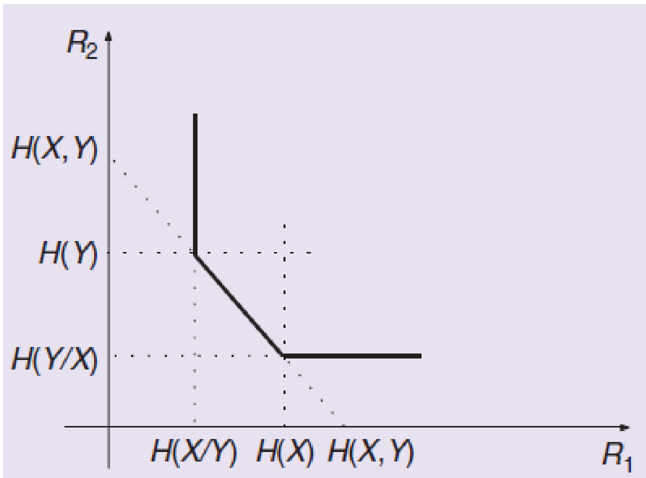
- Neccessary in digital sensor network architectures
- Multiple correlated sources (e.g. X and Y correlated)
- No connection between Encoder 1 and 2 (distributed)
- X and Y should be recovered perfectly from the decoder even if they are correlated
- Same compression rate as in the uncorrelated case should be acheived



Distributed source coding (2)

- Slepian Wolf Theorem
- Specifies the set of rates that allow the decoder to reconstruct these correlated data streams with arbitrarily small error probability
- If the rates at wich X and Y had been encoded satisfy

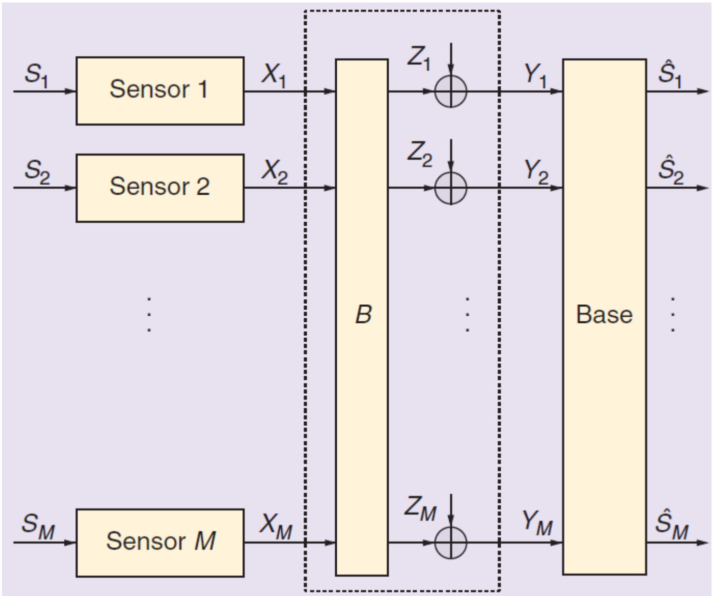
$$R_1 \geq H(X|Y), \quad R_2 \geq H(Y|X), \quad R_1 + R_2 \geq H(X,Y)$$



- Extension:
  Lossy coding of distributed sources also possible
  → Wyner and Ziv

Digital Architectures, 1st scenario (1)

- Expanding Sensor Network
  - M sources $S_{1...M}$, M sensors, M channels with channel noises Z
  - $S_{1...M}$ are independent of each other
  - Rich Communication infrastructure: MIMO Channel matrix B is full rank (a diagonal matrix in the wired case), channel noise $Z_{1...M}$
  - The basestation wishes to recover all cont. time sensor readings



Quality measure for recovering sensor readings:

Distortion measure D:

$$D = \frac{1}{M} \sum_{m=1}^{M} E\left[\left\| S_m - \hat{S}_m \right\|^2 \right]$$

Digital Architectures, 1st scenario (2)

- Our Goal: relationship between D and the source characteristics and the communication infrastructure (e.g. P_{TOT}…total power cosumed by sensors)
- Assumptions:
  - $S_{1...M}$ are iid Gaussian RV's with zero mean and unit variance
  - B is an identity matrix
  - Result:

$$D = \frac{M + \sigma_Z{}^2}{P_{TOT}} \qquad D(M) \propto \frac{M}{P_{TOT}}$$

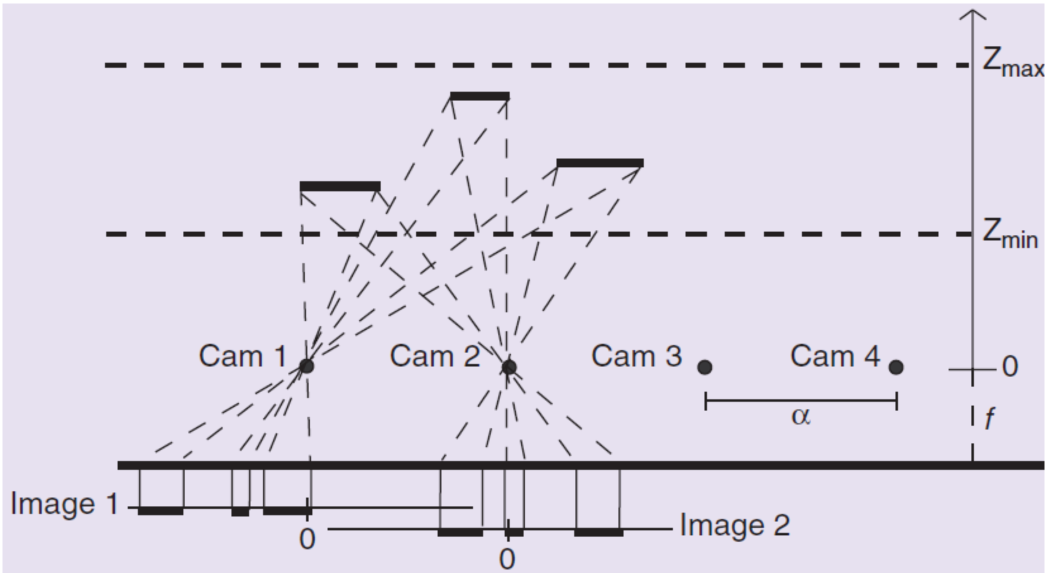
  - M parallel channels
  - D can be acheived by a separate design for source and channel codes

Digital Architectures, 2nd scenario

- Camera Sensor Network example
  - M digital pinhole cameras located along a line, L Lambertain planes
  - Cameras communicate to a BS trough a Multiacess channel with the capacity C = ½ ld(1 + (P_{TOT}/σ^2))
  - Each camera observes a perspective projection of the visual scene
  - Each camera observes a blurred and sampled version of the original projection, there in many cases exact reconstruction is possible
  - Scaling behaviour:

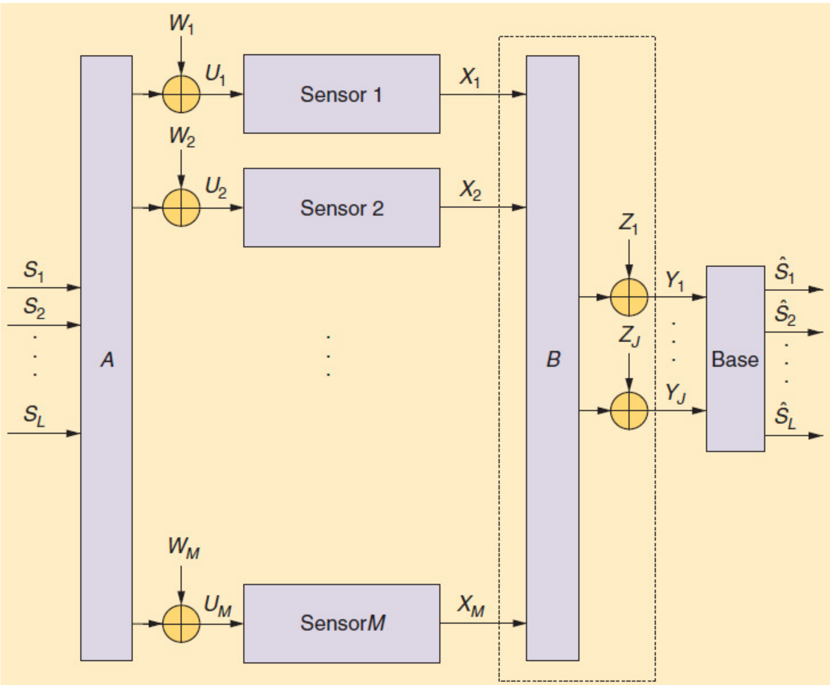
$$D(M) \propto \frac{1}{(P_{TOT}(M))^{\gamma}}$$

$$D(M)_{LOWERBOUND} \propto \frac{1}{M(P_{TOT}(M))^{\gamma}}$$



Digital Architectures, 3rd scenario (1)

- Refining Sensor Network
  - L sources, M sensors → each sensor picks up a merged version of the underlying sources (L < M)
  - Observation noise: $W_{1...M}$, Channel noise: $Z_{1...M}$
  - B has low rank: Poor communication infrastructure (J channels to the BS)



Quality measure for recovering sensor readings:

Distortion measure D:

$$D = \frac{1}{L} \sum_{\ell=1}^{L} E\left[\left\| S_\ell - \hat{S}_\ell \right\|^2 \right]$$

Digital Architectures, 3rd scenario (2)

- Our Goal: relationship between D and the source characteristics and the communication infrastructure (e.g. P_{TOT}…Total power cosumed by sensors)
- Quadratic gaussian CEO Problem[2] (Central Estimating Officier)
- L = 1, J = const.
- S is a gaussian distribution

$$R^{\mathrm{CEO}}(D) = \log_2\left(\frac{\sigma_S^2}{D}\right) + M\log_2\left(\frac{M\sigma_S^2}{M\sigma_S^2 - \sigma_W^2\left(\frac{\sigma_S^2}{D} - 1\right)}\right)$$

- Distortion behaves as:

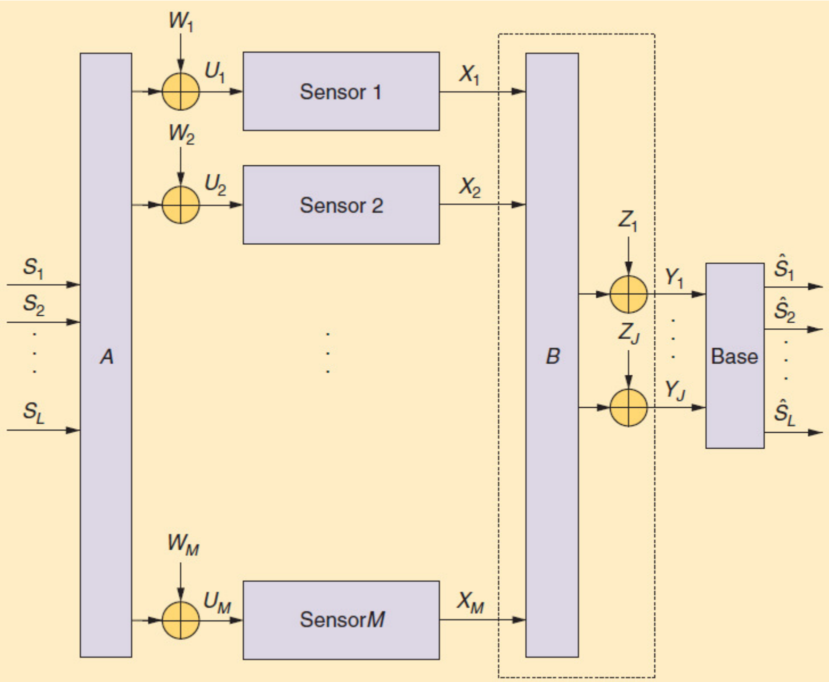
$$D_{\mathrm{digital}}(M) \sim \frac{1}{\log(MP_{\mathrm{tot}}(M))}$$

Analog Architectures

- Way of coding which is not based on bit streams
- Joint source channel coding
- No general tools → case-by-case designs
- Simple architecture:
  - Each sensor scales it‘s noisy observation by an appropriately chosen factor and transmits this on the channel
  - Strong inference between the sensors
  - The interference is designed so that it results in a cooperation gain

Analog Architectures, scenario

- Refining sensor network
  - L = J = 1
  - $A = B^T = (1,1,\ldots,1)$
  - S = iid gaussion RV‘s, zero mean variance σ_S^2



$$Y[n] = Z[n] + \sum_{m=1}^{M} X_m[n]$$

$$D = \sigma_S^2 - \frac{(E[SY])^2}{E[Y^2]}$$

$$D_{\text{analog}}(M) \sim \frac{1}{M P_{\text{tot}}(M)}$$

Comparsion Analog vs. Digital (1)

- When the separation theorem does not hold the gap between the two architectures is increasing exponential
- For the refining sensor Network:

$$D_{\text{analog}}(M) \sim \frac{1}{MP_{\text{tot}}(M)} \qquad D_{\text{digital}}(M) \sim \frac{1}{\log(MP_{\text{tot}}(M))}$$

$$M_{\text{digital}} \approx e^{M_{\text{analog}}}$$

- The digital architecture needs exponentially more sensors
- The question of how much information is aquired by a sensor network cannot generally expressed in terms of bits!

Comparsion Analog vs. Digital (2)

- The amount of information depends on the overall structure of the sensor network
- But the previous results should not mean that pure analog transmission should be the best possible strategy

Conclusions and challenges

- While digital might be convenient, analog might be optimal
- Expanding Network example
  - Separation theorem holds
- Refining Sensor Network
  - Observations are noisyless (camera network example)
    - Small penalty doing separation
    - An approximate (scaling-law) separation theorem holds
  - Noisy observations
    - Separation theorem does not hold → creative designs
- Fully general solution still open

  (are there multiuser joint source-channel code that could reap some of the exponential gain?)

  → Future challenge to find it

References

[1] Sensing reality and communicating bits: A dangerous liaison Gastpar, M.; Vetterli, M.; Dragotti, P.L., Signal Processing Magazine, Vol. 23, No. 4, 2006.

[2] T. Berger, Z. Zhang, and H. Viswanathan, "The CEO problem," IEEE Trans. Inform. Theory, vol. IT–42, pp. 887–902, May 1996.

[3] T. Ajdler, L. Sbaiz, and M. Vetterli, "The plenacoustic function and its sampling," IEEE Trans. Signal Processing, to be published.