Signaltransformationen SPSC

15.03.2013

1a)

$$x[n] = \sum_{k=0}^{N_0-1} a_k \cdot e^{+j\frac{2\pi}{N_0}kn} = \sum_{k=0}^{3} e^{j\frac{\pi}{2}kn}$$ $\frac{1}{2}$

$$= 1 + e^{j\frac{\pi}{2}n} + (-1)^n + e^{-j\frac{\pi}{2}n} = 1 + (-1)^n + 2\cdot\frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$$ $\frac{1}{2}$

$$= 1 + (-1)^n + 2\cos\left(\frac{\pi}{2}n\right)$$

$x[0] = 2 + 2\cdot 1 = 4$ $\qquad$ $x[2] = 2 - 2\cdot(-1) = 0$

$x[1] = 0 + 2\cdot 0 = 0$ $\qquad$ $x[3] = 0 + 2\cdot 0 = 0$ 1

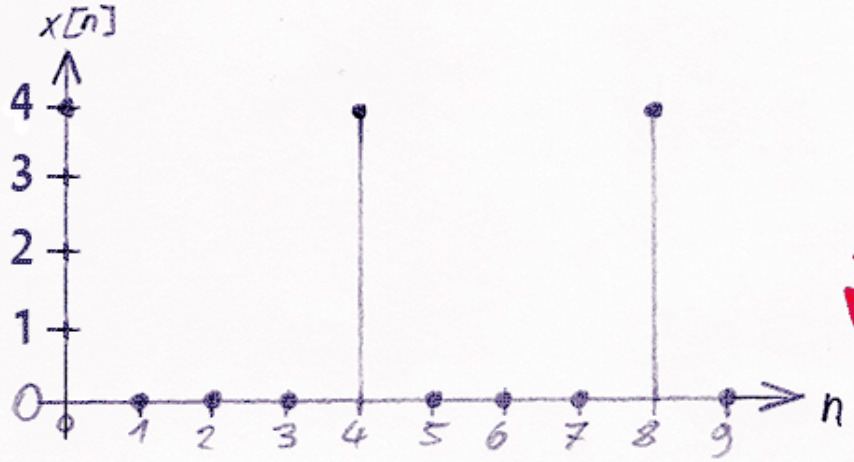
1a Alternative)

$$a_k = 1 = e^0 \overset{!}{=} \frac{1}{N_0}\cdot\sum_{n=0}^{N_0-1} x[n]\cdot e^{-j\frac{2\pi}{N_0}kn} = \frac{1}{4}\cdot\sum_{n=0}^{3} x[n]\cdot e^{-j\frac{\pi}{2}kn}$$ $\frac{1}{2}$

$$e^0 = \frac{1}{4}\Big(\underbrace{x[0]}_{=4}\cdot e^0 + \underbrace{x[1]}_{=0}\cdot e^{-j\frac{\pi}{2}k} + \underbrace{x[2]}_{=0}\cdot e^{-j\pi k} + \underbrace{x[3]}_{=0}\cdot e^{-j\frac{3\pi}{2}k}\Big)$$ $\frac{1}{2}$

Koeffizientenvergleich: 1

1b)



1

2a) $x(t) = \cos\left(\frac{t}{2}\right) + \cos\left(\frac{t}{3}\right)$

$\omega_1 = \frac{1}{2} \Rightarrow T_1 = \frac{2\pi}{\omega_1} = 4\pi \qquad \omega_2 = \frac{1}{3} \Rightarrow T_2 = \frac{2\pi}{\omega_2} = 6\pi$

$T_0 = \mathrm{kgV}(4\pi, 6\pi) = 2\pi \cdot \mathrm{kgV}(2,3) = 12\pi$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{12\pi} = \frac{1}{6}$ 1

2b) $x(t) = \cos\left(3 \cdot \frac{1}{6} t\right) + \cos\left(2 \cdot \frac{1}{6} \cdot t\right) = \cos(3\omega_0 t) + \cos(2\omega_0 t)$ $\frac{1}{2}$

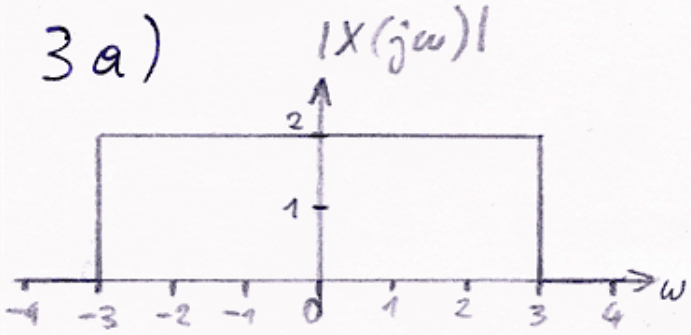
$= \frac{1}{2} e^{j3\omega_0 t} + \frac{1}{2} e^{-j3\omega_0 t} + \frac{1}{2} e^{j2\omega_0 t} + \frac{1}{2} e^{-j2\omega_0 t}$ $\frac{1}{2}$

$\overset{!}{=} \sum_{k \in \mathbb{Z}} a_k \cdot e^{jk\omega_0 t}$ $\frac{1}{2}$

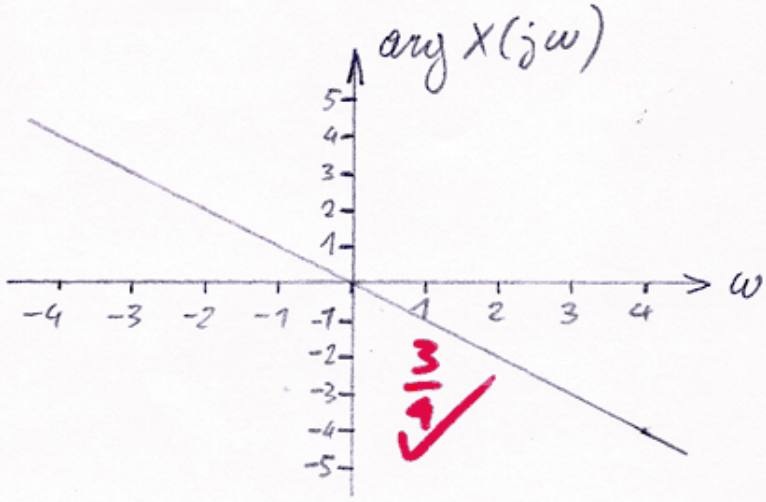
Koeffizientenvergleich:

$$a_k = \begin{cases} \frac{1}{2} & k \in \{-3, -2, 2, 3\} \\ 0 & \text{sonst} \end{cases}$$ $\frac{1}{2}$

3 a)



$\frac{3}{4}$



$\frac{3}{4}$

3b über inv. FT) Aus $X(j\omega) = |X(j\omega)| \cdot e^{j \arg X(j\omega)}$ folgt

$$X(j\omega) = \begin{cases} 2 \cdot e^{-j\omega} & \text{wenn } -3 \le \omega \le 3 \\ 0 & \text{sonst} \end{cases}$$ $\frac{1}{2}$

$$x(t) = \frac{1}{2\pi} \cdot \int_{\mathbb{R}} X(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi} \cdot 2 \cdot \int_{-3}^{3} e^{-j\omega} e^{j\omega t}\, d\omega = \frac{1}{\pi} \cdot \int_{-3}^{3} e^{j\omega(t-1)}\, d\omega$$ $\frac{1}{2}$

$$= \frac{1}{\pi} \frac{1}{j(t-1)} \left(e^{j\omega(t-1)} \Big|_{\omega=-3}^{\omega=3} \right) = \frac{2}{\pi(t-1)} \cdot \frac{1}{2j} \left(e^{j3(t-1)} - e^{-j3(t-1)} \right)$$ $\frac{1}{3}$ $\frac{1}{3}$

$$= \frac{2}{\pi} \cdot \frac{\sin(3(t-1))}{t-1} \in \mathbb{R}$$ $\frac{1}{3}$

3 b über Zeitverschiebungssatz)

Definition Rechteckfunktion:

$$\mathrm{rect}(\omega) := \begin{cases} 1 & \text{wenn } -\frac{1}{2} \le \omega \le \frac{1}{2} \\ 0 & \text{sonst} \end{cases}$$

Mit $a > 0$ gilt:

$$\mathrm{rect}\left(\frac{\omega}{2a}\right) \bullet\!\!-\!\!\circ \frac{\sin(at)}{\pi t}$$

Damit ergibt sich $X(j\omega) = 2 \cdot \mathrm{rect}\left(\frac{\omega}{6}\right) \cdot e^{-j\omega}$ $\frac{1}{3}$

(siehe oben)

$$\mathrm{rect}\left(\frac{\omega}{6}\right) = \mathrm{rect}\left(\frac{\omega}{2 \cdot 3}\right) \bullet\!\!-\!\!\circ \frac{\sin(3t)}{\pi t}$$ $\frac{1}{2}$

(konstanter Faktor)

$$2 \cdot \mathrm{rect}\left(\frac{\omega}{2 \cdot 3}\right) \bullet\!\!-\!\!\circ 2 \cdot \frac{\sin(3t)}{\pi t}$$ $\frac{1}{3}$

(Zeitverschiebungssatz)

$$X(j\omega) = 2 \cdot \mathrm{rect}\left(\frac{\omega}{2 \cdot 3}\right) \cdot e^{-j\omega} \bullet\!\!-\!\!\circ 2 \cdot \frac{\sin(3(t-1))}{\pi(t-1)} = x(t)$$ 1