Homework Assignment 6

Name(s) Matr.No(s).

The MATLAB programs and the simulation protocol of this homework have to be submitted via e-mail to `hw2.spsc@tugraz.at` no later than **2011/5/18**. The subject of the e-mail consists of your last name(s) and the assignment number (e.g., "Farnsworth, Zoidberg: Assignment 6"). Make sure your programs contain helpful documentation.

MATLAB Problem 6.1 (10 Points)

In this problem we will design and evaluate a low-rate, constrained-resolution VQ. We consider the squared-error distortion criterion and the jointly Gaussian (vector) variable $\mathbf{X}$ with zero mean and the covariance matrix

$$C_{\mathbf{X}} = \begin{bmatrix} 1.25 & 0.75 \\ 0.75 & 1.25 \end{bmatrix}.$$

(a) Write the MATLAB function `[CB, D_it] = lloyd( X, N_cells, N_it, CB_init)` that implements the Lloyd training algorithm for constrained-resolution VQs. The matrix `X` contains samples of a vector source with arbitrary dimensionality, `N_cells` is the number of cells (number of codebook entries), `N_it` is the (maximum) number of iterations, `CB_init` is an (optional) initial codebook (i.e., initial centroids), `CB` is the optimized codebook, and `D_it` is a vector containing the overall mean distortion evaluated after each iteration.

(b) Generate 100 000 realizations of the above given vector source to be used as training data. Design VQs for rates between 5 and 15 bits (in 2-bit steps) using the previously implemented Lloyd algorithm. Initialize the centroids uniformly over a rectangular area including all training samples. Do not compute too many iterations (max. 50, otherwise your computer may be busy for a while). On the other hand, compute enough iterations to obtain convergence (double-check using the distortion-over-iteration graph).

(c) As an alternative approach, we use a KLT[1] followed by two low-rate, constrained-resolution SQs (and the inverse KLT in the decoder). (i) Write down the KLT for the given source and write down the covariance matrix of the transformed variable. (ii) For a (total) rate between 5 and 15 bits (in 2-bit steps), compute the optimal rate allocation (i.e., apply reverse waterfilling) among the two SQs. (iii) Apply the transform to your training data and use your implementation of the Lloyd algorithm to train the two SQs for all considered rates (codebook sizes).

(d) Evaluate the performance of the obtained VQs from **(b)** as well as the transform-based approach from **(c)** using separately generated test data (i.e., another 100 000 realizations of the given random variable). Plot the (total) rate/distortion pairs in one plot.

[1] The transform that diagonalizes the covariance matrix.

MATLAB Problem 6.2 (4 Points)—Bonus Problem: LBG Algorithm

Write the MATLAB function `[CB, D_it] = lloyd_lbg( X, N_cells, N_it)` that incorporates the LBG initialization and codebook-doubling idea into the Lloyd training algorithm. You may implement a convergence check that automatically aborts the Lloyd algorithm. For two-dimensional training data, visualize how the centroids are added and optimized. Plot the distortion-over-time graph[2].

[2] This graph should exhibit jumps whenever the codebook size is increased