

Homework Assignment 4

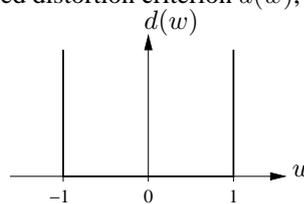
Name(s)

Matr.No(s)

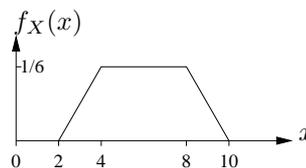
Your homework has to be delivered as hard copy at the beginning of the lecture on **2011/4/13**. Use a printed version of this assignment document as the title page(s) and fill in your name(s) and matriculation number(s). You may work in pairs (only one copy needs to be delivered per pair).

Analytical Problem 4.1 (6 Points)

Consider the following distance-based distortion criterion $d(w)$, where $w = x - \hat{x}$:



- (a) Derive the Shannon lower bound $R_{SLB}(D)$ for this criterion.
- (b) Show whether the Shannon lower bound is tight¹ or not when the source X has
1. a Gaussian distribution with mean μ and variance σ^2 ,
 2. a uniform distribution on the interval $|x| < a$, or
 3. the following trapezoidal density:



Problem 4.2 (5 Points)

Consider the two independent random variables X_1 and X_2 with Laplacian probability density functions $f_{X_i}(x) = \frac{a_i}{2} e^{-a_i|x|}$. In the following, use $a_1 = 1/4$ and $a_2 = 3/4$. The absolute-error criterion $D_i = E\{|X_i - \hat{X}_i|\}$ applies. The overall distortion and the overall rate are the sums $D = D_1 + D_2$ and $R = R_1 + R_2$, respectively.

- (a) Write down and plot the rate-distortion functions of the individual variables $R_i^*(D_i)$. (You may copy the results from examples of Chapter 6.)
- (b) Find a set of equations that determines the overall rate-distortion function $R^*(D)$. (The solution for Gaussian variables, known as “reverse water filling,” has not yet been shown to apply here...)
- (c) Write a computer program that computes and plots (i) the overall rate-distortion function $R^*(D)$ as well as (ii) the optimal rate distribution as a function of the overall rate², $R_1(R)$ and $R_2(R)$. Append the plot(s) to your homework and discuss your results.

¹I.e., whether it is equal to the rate-distortion function $R^*(D)$.

²Use a total rate R between 0 and 6bits.