Homework Assignment 2

Name(s) Matr.No(s).

Your homework has to be delivered as hard copy at the beginning of the lecture on **2011/3/25**. Use a printed version of this assignment document as the title page(s) and fill in your name(s) and matriculation number(s). You may work in pairs. Only one copy needs to be delivered per pair.

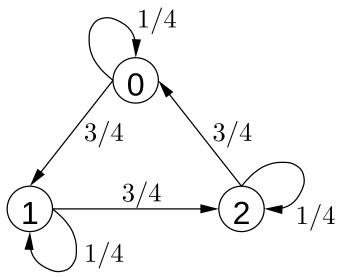
Please, make sure that your approaches, procedures, and results are clearly presented. Justify your answers.

Analytical Problem 2.1 (5 Points)

Solve Problem 7 (a) and (b) of Chapter 4 in our lecture notes (page 84).

Analytical Problem 2.2 (5 Points)

Consider the stationary Markov process $X_n \in \{0, 1, 2\}$ with the following chain and state-transition probabilities:



(a) Consider a memoryless, sample-by-sample encoding of the state sequence, i.e., consider the random variable $X = X_n$. Determine the probabilities $p_X(x)$ and compute $H(X)$.

(b) Let us encode blocks of two consecutive samples of X_n. We denote the obtained random variable that describes the blocks as Y. Compute the probabilities $p_Y(y)$ and compute $H(Y)$ as well as $H_2(X_n)$.

(c) Similarly as above, let us encode blocks of three consecutive samples of X_n. We denote the obtained random variable that describes the blocks as Z. Compute the probabilities $p_Z(z)$ and compute $H(Z)$ as well as $H_3(X_n)$.

(d) Find a computationally efficient encoder/decoder pair that exploits the dependencies between the samples (the structure of the given process is known; ignore the problem of initializing the encoder/decoder; blocks of arbitrary length may be transmitted). Derive the lower bound on the average bit rate of such an encoding approach.

Analytical Problem 2.3 (4 Points)—Bonus Problem

(a) Starting at the definition of *Conditional Entropy*

$$H(Y|X) = -E\left\{\log_2(p_{Y|X}(Y|X))\right\},$$

show that

$$H(Y|X) = H(X,Y) - H(X).$$

(b) Show that

$$H(Y|X) \leq H(Y),$$

i.e., conditioning reduces entropy. In what case are both sides equal?

(c) Express the following *Conditional Mutual Information*

$$I(X;Y|Z)$$

in terms of conditional entropies. Visualize $I(X;Y|Z)$ as well as the obtained conditional entropies by a 'bubble diagram.'