Homework Assignment 1

Name(s) Matr.No(s).

Your homework has to be delivered as hard copy at the beginning of the lecture on 2011/3/16. Use a printed version of this assignment document as the title page(s) and fill in your name(s) and matriculation number(s). You may work in pairs. Only one copy needs to be delivered per pair. Please, make sure that your approaches, procedures, and results are clearly presented. Justify your answers.

Analytical Problem 1.1 (7 Points)

Consider the random variable \( X \) with the symbol set \( \mathcal{A} = \{a, b, c, d\} \) with symbol probabilities 0.05, 0.05, 0.2, and 0.7.

(a) Compute \( H(X) \).

(b) Give the length of the shortest possible equal-length code for \( X \).

(c) Construct a Shannon prefix code\(^1\) for \( X \). Evaluate the performance of this code (i.e., calculate the average codeword length).

(d) Let us encode blocks of 2 symbols of \( X \) by assigning a single codeword to each block (or ‘super symbol’). Denote the new random variable obtained by concatenation as \( Y \). Compute \( p_Y(y) \), i.e., the probabilities of all symbol pairs.

(e) Compute \( H(Y) \) and show its relation to \( H(X) \).

(f) Compute the average codeword length for a Shannon prefix code for \( Y \).

(g) Let us encode blocks of 3 symbols of \( X \) by assigning a single codeword to each such block. Denote the new random variable obtained by concatenation as \( Z \). Compute \( p_Z(z) \) and the average codeword length for a Shannon prefix code for \( Z \).

(h) Visualize the performance of the four coding approaches (b), (c), (f), and (g) as well as \( H(X) \) on an axis with the unit ‘average bits per symbol of \( X \).’

Analytical Problem 1.2 (3 Points)

(a) Show that

\[
\log(x) \leq x - 1, \quad \forall x > 0.
\]

\(^1\)Use Shannon’s rule for codeword lengths.