Fundamentals of Digital Communications
Problem Class 6
Random Processes

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Problem 6.1
A zero-mean white noise process \( n(t) \) with double-sided power spectral density (PSD) \( N_0/2 \) is passed through a linear filter that is characterized by an impulse response \( h(t) \).
(a) Find the autocorrelation function and PSD of the output signal \( x(t) \) of the filter.
(b) Compute the first and second-order moments of the filter output (mean value and variance).

Problem 6.2
A transmitted (baseband) signal with PAM, PSK, or QAM modulation can be expressed as
\[
v(t) = \sum_{n=-\infty}^{\infty} a_ng_T(t-nT)
\]
where \( \{a_n\} \in \mathbb{C} \) represents the (PAM, PSK, or QAM modulated) information symbols, \( g_T(t) \) describes the pulse shape, \( n \) is a symbol index, and \( T \) is the symbol interval.
(a) The signal \( v(t) \) can be seen as a sample function of a (continuous-time) random process, because the symbol sequence \( \{a_n\} \) is a random sequence representing the information sequence. Determine the power spectral density (PSD) of the random process.
(b) Consider the information sequence \( \{a_n\} \) is constructed from a binary antipodal modulation scheme (i.e. \( a_n \in \{\pm1\} \)) and its symbols are uncorrelated. Again, determine the PSD of the random process.
(c) Compute the PSD, when \( g_T(t) \) is a rectangular pulse
\[
g_T(t) = \begin{cases} A & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}
\]
(d) Compute the PSD, when \( \{a_n\} \) represents (binary) OOK modulation, i.e. \( a_n \in \{0, 1\} \).