Fundamentals of Digital Communications
Problem Class 5
Probability and Random Variables

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Problem 5.1

Consider signal transmission over a so-called "erasure channel". The transmitted binary symbols are described by mutually exclusive events A_1 and A_2, ocurring with prior probabilities $P(A_1) = 0.3$ and $P(A_2) = 0.7$. The receiver output is quantized to three discrete observations, denoted by events B_i, $i = 1, 2, 3$, as illustrated by the figure. B_2 is the erasure event, implying an uncertainty upon the transmitted symbol, while certain decisions can be made, when B_1 or B_3 are observed. The channel is characterized by the following conditional probabilities: $P(B_2|A_1) = 0.3$ and $P(B_2|A_2) = 0.2$.

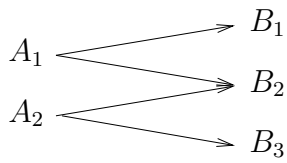

Figure 1: Erasure channel.

(a) Determine for the observation B_2 the decision for the symbol A_k that has been transmitted with higher probability. This decision will minimize the average probability of making a decision error.
(b) Find the average error probability for this case and compare it with the error probability when the opposite decision were made.

Problem 5.2

The voltages of a set of batteries are modeled by a continuous Gaussian random variable X with mean value of $m = 1.5$ V and standard deviation of $\sigma = 0.1$ V, if classified as *"good"*. If classified as *"broken"*, their voltage is modeled as 0 V. Furthermore, we know the probability $P(\text{good}) = \sqrt{2\pi}/10 \approx 0.25$.

(a) Sketch to scale the probability density function (PDF) and the cumulative distribution function (CDF) of the random variable X.

(b) Determine the probability that a battery has a voltage between $1.4 < X < 1.6$ V.

Problem 5.3

The error introduced in a quantization operation (quantization noise) has to be characterized statistically. We write the quantization of the input variable X as

$$Y = \text{round}(X)$$

leading to the error signal $E = Y - X$. Furthermore, we know from the input variable X that it has a continuous PDF and a standard deviation $\sigma_X \gg 1$.

(a) Find and sketch the PDF and CDF of the quantization error E.

(b) Determine the power (= variance) of the quantization noise.