Fundamentals of Digital Communications
Class 2: Geometric Representation of Signal Waveforms

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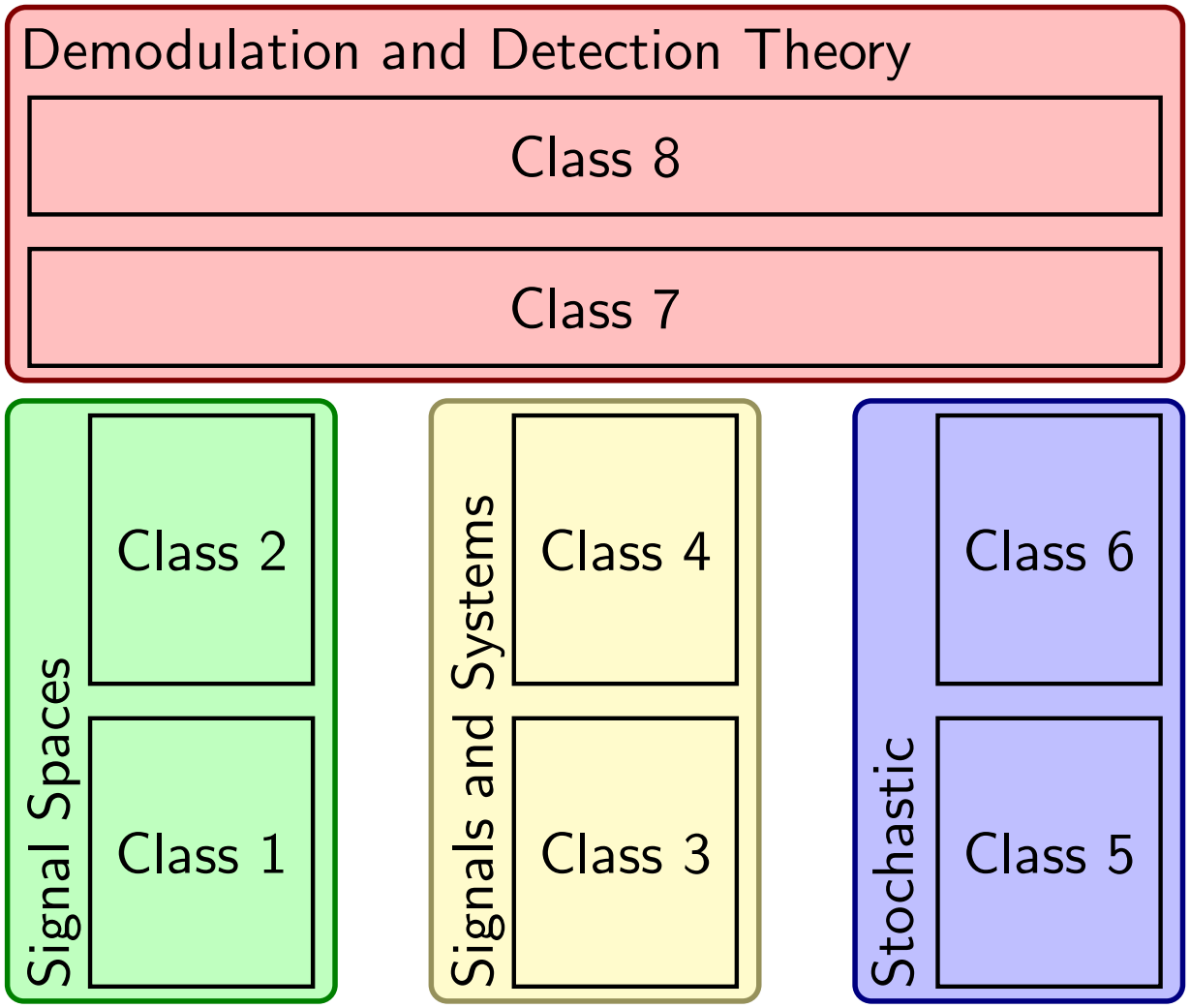
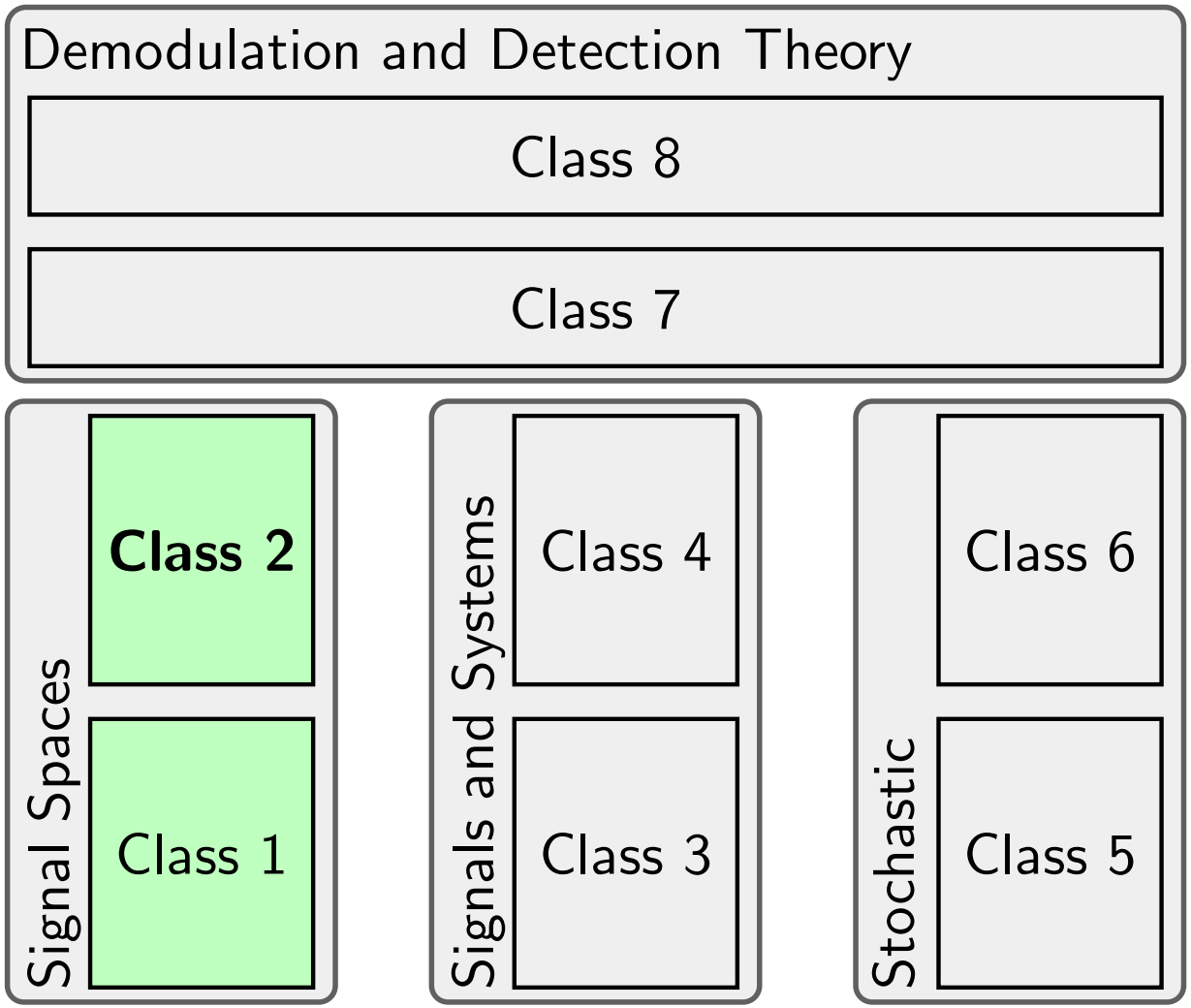
Schedule – to master Digital Communications



Schedule – to master Digital Communications



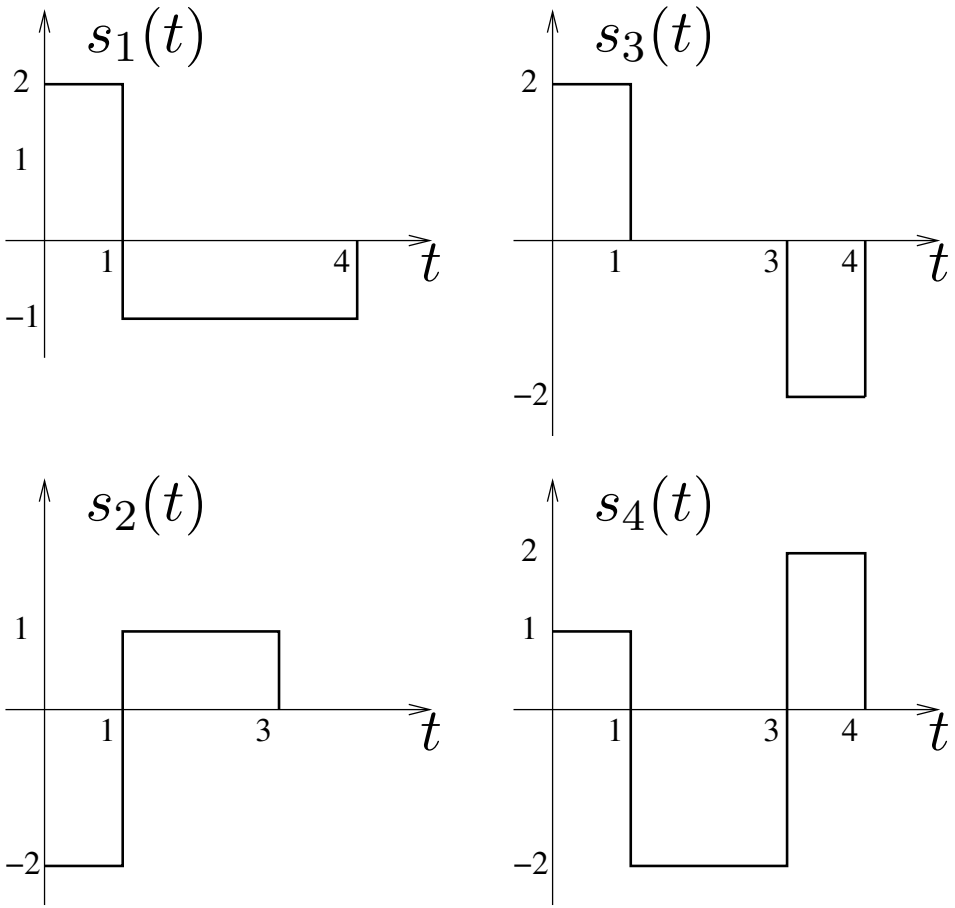
Class 2 – Geometric Representation of Signal Waveforms

Goals:

- understand Gram-Schmidt-orthogonalization
- understand subspaces and the projection onto them
- understand signal and vector spaces (a little geometry)
- compute the distance between two signal waveforms

Problem 2.1

Given:



Problem 2.1 (cont'd)

Question:

- find coefficient vectors $\mathbf{p}_i$ in 4-D space $\mathcal{P} = \mathrm{span}\{\phi_1(t), \ldots, \phi_2(t)\}$
- use Gram-Schmidt orthogonalization to find dimensionality N and basis $\mathbf{q}_j$, $j = 1, \ldots, N$ for $\mathcal{S} = \mathrm{span}\{\mathbf{p}_1, \ldots, \mathbf{p}_4\}$
- sketch corresponding orthonormal basis functions $\{\psi_j(t)\}_{j=1}^{N}$ for $\mathcal{S} = \mathrm{span}\{s_1(t), \ldots, s_4(t)\}$
- determine $\mathbf{s}_i$ for signal space $\mathcal{S}$
- distance between signals $s_1(t)$ and $s_2(t)$:
  1. directly,
  2. from their vectorial representation in $\mathcal{P}$, and
  3. from their vectorial representation in $\mathcal{S}$.

Problem 2.2

Given:

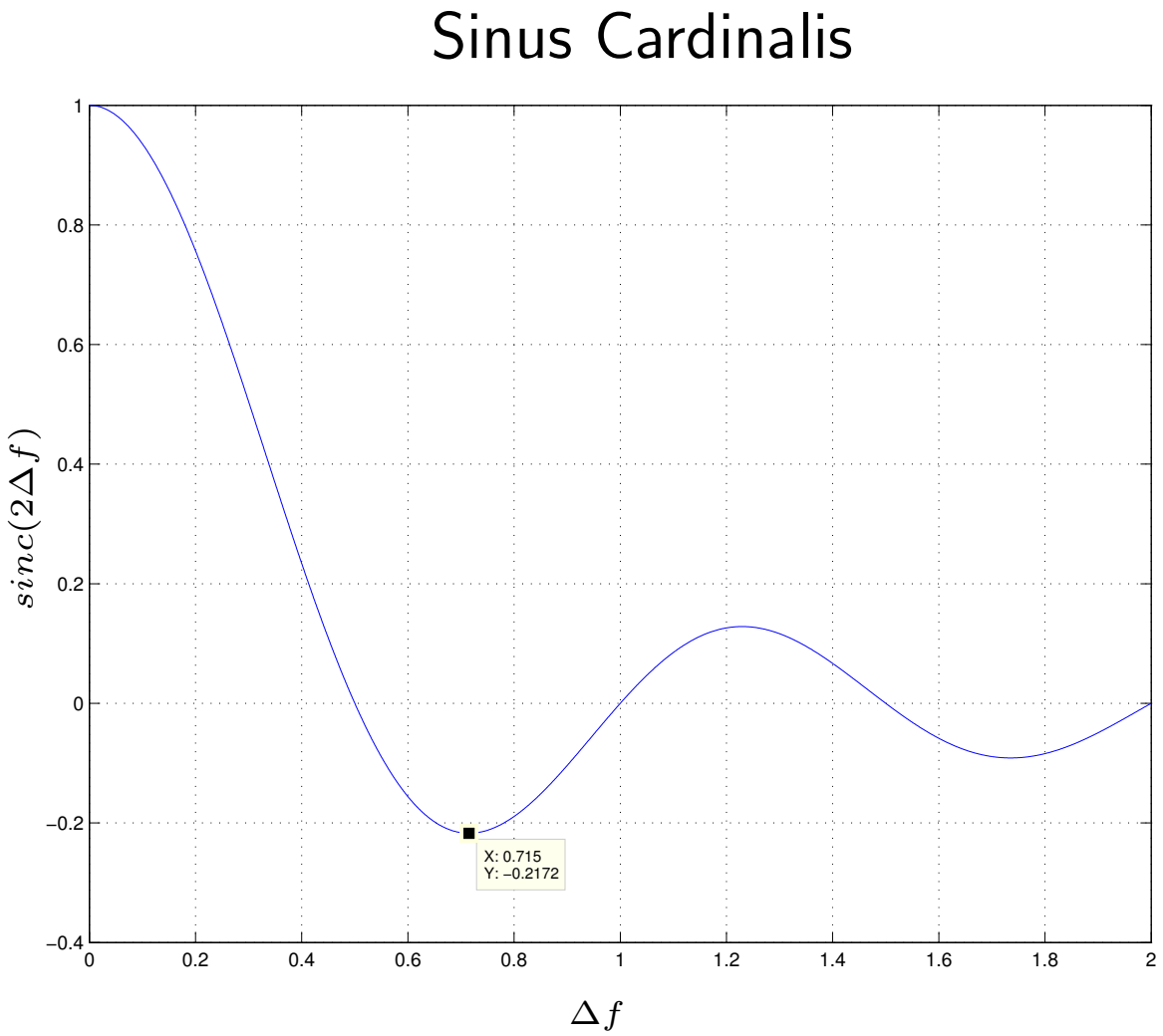
$$u_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_1 t) \qquad 0 \leq t \leq T$$

$$u_2(t) = \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_2 t) \qquad 0 \leq t \leq T$$

Question:

- Find and maximize the distance between the waveforms $d = \|u_2(t) - u_1(t)\|$ as a function of the frequency spacing $\Delta f = f_2 - f_1$. Assume $f_1,\ f_2 \gg 1/T$

Problem 2.2


Sinus Cardinalis
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$sinc(2\Delta f)$
Δf
X: 0.715
Y: −0.2172

Questions

- How can you determine the dimension of a signal space?
- What does the projection theorem state?
- Explain the Gram-Schmidt procedure?

Questions

- How can you determine the dimension of a signal space? via Gram-Schmidt
- What does the projection theorem state? projection error is orthogonal to subspace
- Explain the Gram-Schmidt procedure?
  1. Choose one vector, normalize it $\rightarrow$ first basis.
  2. Project next vector onto current basis. If projection error is zero, no new basis. Otherwise normalize projection error (orthogonal to current basis) $\rightarrow$ next basis.
  3. Repeat step 2 for all remaining vectors.

Orthonormal Basis (cont'd)

Gram-Schmidt orthogonalization

- find the $N \leq M$ orthonormal basis functions
  - $\psi_1(t) = \frac{s_1(t)}{\|s_1(t)\|}$

    $\mathcal{S}_1 = \text{span}\{\psi_1(t)\}$
  - $\psi_2(t) = \frac{s_2(t) - \hat{s}_2(t)}{\|s_2(t) - \hat{s}_2(t)\|}$, $\quad \overbrace{\hat{s}_2(t) = c_{21}\psi_1(t)}^{\text{projection of } s_2(t) \text{ onto } \mathcal{S}_1}$

    $\mathcal{S}_2 = \text{span}\{\psi_1(t), \psi_2(t)\}$
  - $\vdots$
  - $\psi_k(t) = \frac{s_k(t) - \hat{s}_k(t)}{\|s_k(t) - \hat{s}_k(t)\|}$, $\quad \overbrace{\hat{s}_k(t) = \sum_{i=1}^{k-1} c_{ki}\psi_i(t)}^{\text{projection of } s_k(t) \text{ onto } \mathcal{S}_{k-1}}$

    no basis function, if $s_k(t) - \hat{s}_k(t) = 0$