Problem 2.1
Signal Space Representations.

(a) Given the set of orthonormal basis functions

\[ \phi_i(t) = \begin{cases} 
1 & (i-1) \leq t \leq i \\
0 & \text{elsewhere}
\end{cases} \]
for \( i = 1, 2, 3, 4 \), express the waveforms of Signal Set 1 in the 4-dimensional signal space 
\( \mathcal{P} = \text{span}\{\phi_1(t), ..., \phi_4(t)\} \) as vectors \( \mathbf{p}_i \).

(b) Using the Gram-Schmidt Orthogonalization, determine the dimensionality \( N \) and find an 
orthonormal basis \( \mathbf{q}_j, j = 1, ..., N \) for these signal vectors; i.e. the dimensionality of the signal 
space \( \mathcal{S} = \text{span}\{\mathbf{p}_1, ..., \mathbf{p}_4\} \).

(c) Sketch the corresponding orthonormal basis functions \( \{\psi_j(t)\}_{j=1}^N \) for \( \mathcal{S} = \text{span}\{s_1(t), ..., s_4(t)\} \).

(d) Determine the signal vectors \( \mathbf{s}_i \) for the signal space \( \mathcal{S} \).

(e) Compute the distance between the signals \( s_1(t) \) and \( s_2(t) \):

1. directly,
2. from their vectorial representation in \( \mathcal{P} \), and
3. from their vectorial representation in \( \mathcal{S} \).

**Problem 2.2**

Consider a binary digital communication scheme applying frequency-shift keying (FSK). 
The following waveforms are used:

\[
\begin{align*}
\mathbf{u}_1(t) &= \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_1 t) & 0 \leq t \leq T \\
\mathbf{u}_2(t) &= \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_2 t) & 0 \leq t \leq T 
\end{align*}
\]

(a) Find and maximize the distance between the waveforms, \( d = ||\mathbf{u}_2(t) - \mathbf{u}_1(t)|| \), as a function 
of the frequency-spacing \( \Delta f = f_2 - f_1 \). Assume \( f_1, f_2 \gg 1/T \).