Fundamentals of Digital Communications
Problem Class 2
Geometric Representation of Signal Waveforms

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Problem 2.1

Signal Space Representations.

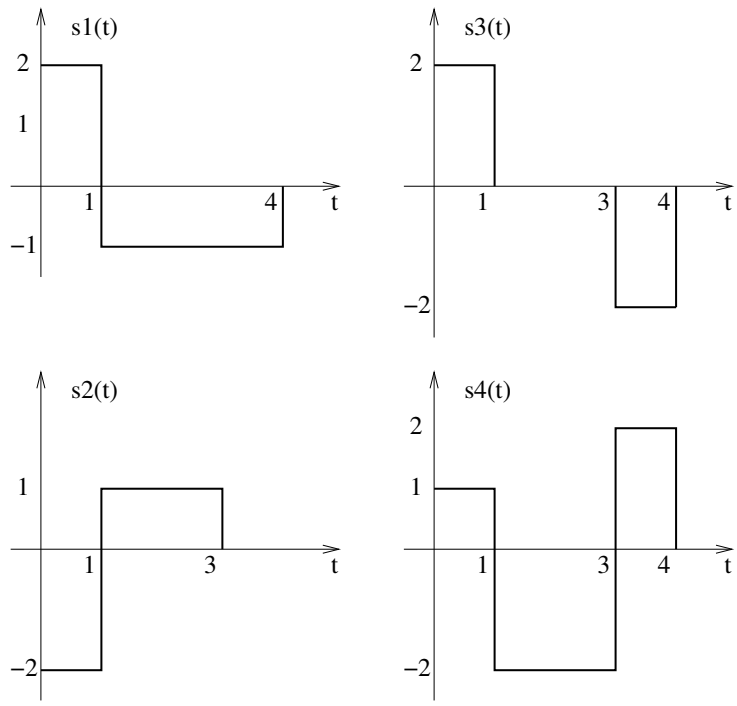

Figure 1: Signal Set 1.

(a) Given the set of orthonormal basis functions

$$\phi_i(t) = \begin{cases} 1 & (i-1) \leq t \leq i \\ 0 & \text{elsewhere} \end{cases}$$

for $i = 1, 2, 3, 4$, express the waveforms of Signal Set 1 in the 4-dimensional signal space $\mathcal{P} = \text{span}\{\phi_1(t), ..., \phi_4(t)\}$ as vectors $\mathbf{p}_i$.
(b) Using the Gram-Schmidt Orthogonalization, determine the dimensionality N and find an orthonormal basis $\mathbf{q}_j$, $j = 1, ..., N$ for these signal vectors; i.e. the dimensionality of the signal space $\mathcal{S} = \text{span}\{\mathbf{p}_1, ..., \mathbf{p}_4\}$.
(c) Sketch the corresponding orthonormal basis functions $\{\psi_j(t)\}_{j=1}^{N}$ for $\mathcal{S} = \text{span}\{s_1(t), ..., s_4(t)\}$.
(d) Determine the signal vectors $\mathbf{s}_i$ for the signal space $\mathcal{S}$.
(e) Compute the distance between the signals $s_1(t)$ and $s_2(t)$:

1. directly,
2. from their vectorial representation in $\mathcal{P}$, and
3. from their vectorial representation in $\mathcal{S}$.

Problem 2.2

Consider a binary digital communication scheme applying frequency-shift keying (FSK). The following waveforms are used:

$$u_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_1 t) \qquad 0 \leq t \leq T$$

$$u_2(t) = \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_2 t) \qquad 0 \leq t \leq T$$

(a) Find and *maximize* the distance between the waveforms, $d = ||u_2(t) - u_1(t)||$, as a function of the frequency-spacing $\Delta f = f_2 - f_1$. Assume $f_1,\ f_2 \gg 1/T$.