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A nonlinear filtered-x prediction error method algorithm for digital predistortion in digital subscriber line systems

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SUMMARY

Digital predistortion of nonlinear systems is an important topic in many practical applications. This paper considers direct predistortion of a Volterra system by connecting in tandem an adaptive Volterra predistorter. The coefficients of the predistorter can be recursively estimated using the nonlinear filtered-x least mean squares (NFxLMS) algorithm. In this paper, the prediction error method (PEM) is used to derive a novel nonlinear filtered-x PEM (NFxPEM) algorithm. A simulation study on Volterra systems shows that the NFxPEM algorithm more significantly suppresses spectral regrowth and converges much faster than the NFxLMS algorithm. Also, the NFxPEM algorithm is used in this paper to design more efficient digital predistorter—as compared with the NFxLMS algorithm—for digital subscriber line systems. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: DSL systems; nonlinear systems; parameter estimation; prediction methods; predistortion; Volterra series

1. INTRODUCTION

In many cases, canceling or reducing the effects of nonlinear distortion is an essential requirement. In wireless communication systems, the nonlinearity of high-power amplifiers is an obstacle in increasing the transfer data rate and mobility. In high-fidelity systems, small distortion produced by nonlinear components dominates the overall performance. Further examples can be found in communication systems, speech processing, and control engineering [1–4].

In [1], three adaptive linearization schemes for weakly nonlinear systems described using Volterra series were proposed. The first linearization scheme estimates the linear and nonlinear subsystems of the physical nonlinear system. Hence, the output of the nonlinear subsystem is evaluated and then subtracted from the output of the physical system. The second and third schemes use a post-processor/preprocessor to postdistort/predistort the signals, respectively. In these cases, necessary estimates of linear and nonlinear operators are provided by adaptive linear and nonlinear filters. The drawback of the first linearization scheme of [1] is that it is hard to perform signal substraction in many practical cases. Also, the second and third schemes require the existence of the inverse of the linear subsystem, which cannot be always guaranteed to be causal and stable.

In [3], a linearization scheme for nonlinear systems was introduced as shown in Figure 1. The idea of the approach is to connect a nonlinear pth-order Volterra predistorter $\boldsymbol{C}_{(p)}$ in tandem with the nonlinear system $\boldsymbol{H}_{(q)}$ that can be described by qth-order Volterra series with M-tap memories and then adaptively adjusting the coefficients of the predistorter in order to reduce the error between

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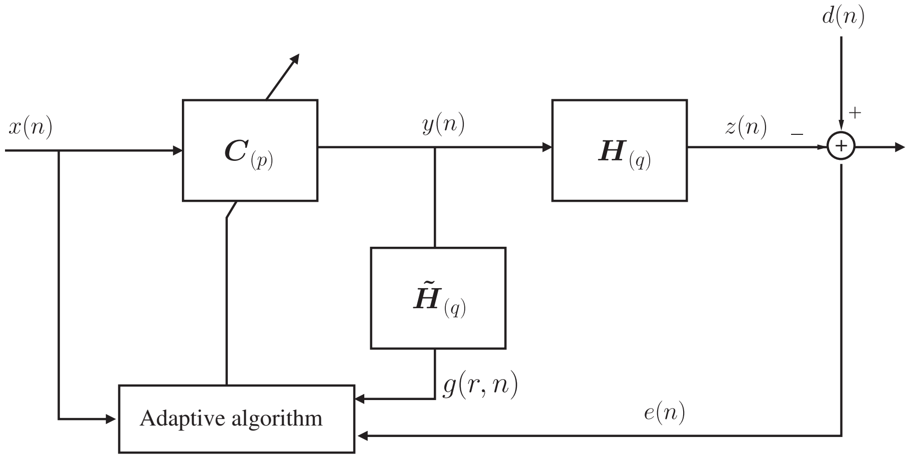

Figure 1. Compensation of nonlinear distortion using nonlinear filtered-x algorithm.

the input and desired signals. These coefficients were estimated recursively using the nonlinear filtered-x least mean squares (NFxLMS) algorithm, which was shown to be reduceable to the linear filtered-x least mean squares (LMS) [5–7], for only the first-order Volterra systems. The approach of [3], like the one introduced in this paper, requires an estimate for the nonlinear system $\boldsymbol{H}_{(q)}$, which is denoted as $\tilde{\boldsymbol{H}}_{(q)}$ in Figure 1 and assumed to be known. Otherwise, a kernel estimation technique for the nonlinear system $\boldsymbol{H}_{(q)}$ based on the adaptive Volterra filter should be considered [8].

In [5], it was shown that the steady-state mean square error of the filtered-x LMS algorithm highly depends on the degree of nonlinearity of the system cascaded with the adaptive filter. Also, the steady-state error increases monotonically with the degree of nonlinearity. Therefore, the NFxLMS algorithm of [3] is expected to provide biased estimates. Also, LMS-type algorithms usually have slow convergence because increasing the step-size parameter leads to instability problems [9].

In this paper, the coefficients of the predistorter are estimated recursively using the recursive prediction error method (RPEM) algorithm [10, 11]. The RPEM algorithm gives consistent parameter estimates under weak conditions in case the asymptotic loss function has a unique stationary point, which represents the true parameter vector [10–12]. Therefore, using the RPEM algorithm is expected to reduce the steady-state mean square error and hence to minimize the total nonlinear distortion at the output of the nonlinear system. Moreover, the RPEM algorithm is known for its high convergence speed.

This paper is organized as follows. In Section 2, a review for the NFxLMS algorithm is given. The nonlinear filtered-x prediction error method (NFxPEM) algorithm is presented in Section 3. The computation complexity (CC) of the NFxPEM algorithm is discussed in Section 4. In Section 5, comparative simulation examples between the NFxPEM and NFxLMS algorithms are given. Conclusions are given in Section 5

2. THE NONLINEAR FILTERED-X LEAST MEAN SQUARES ALGORITHM

The NFxLMS algorithm, introduced in [3] and shown in Figure 1, assumes that the nonlinear system $\boldsymbol{H}_{(q)}$ to be compensated is a discrete time-invariant causal system. The block diagram in Figure 1 consists of the nonlinear physical system $\boldsymbol{H}_{(q)}$ to be compensated using a nonlinear predistorter $\boldsymbol{C}_{(p)}$ and an adaptive algorithm to estimate the proper coefficients of the predistorter. The output of the nonlinear physical system $z(n)$ is compared with the desired output $d(n)$ in order to construct an error signal to be used in the adaptive algorithm in addition to a filtered version from the predistorter's output signal denoted as $g(r,n)$. In Figure 1, the filter used to generate $g(r,n)$ is denoted as $\tilde{\boldsymbol{H}}_{(q)}$ and represents an estimate of the nonlinear physical system $\boldsymbol{H}_{(q)}$. In case the nonlinear

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physical system is already known, $\tilde{\boldsymbol{H}}_{(q)} = \boldsymbol{H}_{(q)}$. In case the nonlinear system is unknown, a system identification method should be used first to identify the system in order to be able to generate $g(r,n)$.

In this paper, the system $\boldsymbol{H}_{(q)}$ with input and output signals $y(n)$ and $z(n)$ can be modeled by qth-order Volterra series with M-tap memories. Hence, the output $z(n)$ is given by

$$z(n) = \sum_{k=1}^{q} \left(\sum_{i_1=0}^{M-1} \cdots \sum_{i_k=0}^{M-1} h_k(i_1,\cdots,i_k) y(n-i_1)\cdots y(n-i_k) \right), \tag{1}$$

where $h_k(i_1,\cdots,i_k)$ are the kth-order kernels of the nonlinear system.

Similarly, the relation between the input and the output of the adaptive Volterra filter $\boldsymbol{C}_{(p)}$ is given by

$$y(n) = \sum_{k=1}^{p} \left(\sum_{i_1=0}^{N-1} \cdots \sum_{i_k=0}^{N-1} c_k(i_1,\cdots,i_k;n) x(n-i_1)\cdots x(n-i_k) \right), \tag{2}$$

where N is the number of memories in the adaptive Volterra filter and $c_k(i_1,\cdots,i_k;n)$ are the kth-order kernels of this filter. According to the pth-order Volterra theorem [13], the Volterra filter $\boldsymbol{C}_{(p)}$ can remove nonlinearities up to the pth-order provided that the inverse of the first-order Volterra system is causal and stable.

The kernels of the adaptive Volterra filter can be estimated by minimizing the mean square distortion defined as

$$\mathrm{E}\{e^2(n)\} = \mathrm{E}\{[d(n) - z(n)]^2\}, \tag{3}$$

where E denotes the expectation and $d(n)$ is the desired signal defined as

$$d(n) = x(n-\tau) + v(n). \tag{4}$$

Here, τ is the time delay necessary to have a causal Volterra predistorter, and $v(n)$ is the zero-mean additive white Gaussian noise.

Remark 1

The delay time τ equals 0 in case the system to be compensated is in minimum phase [3].

The NFxLMS algorithm is obtained by applying the stochastic gradient algorithm [10, 11] as

$$\mathbf{C}_k(n+1) = \mathbf{C}_k(n) - \frac{\mu_k}{2}\, \boldsymbol{\Delta}_k(n), \tag{5}$$

where μ_k is a small positive constant that controls stability and rate of convergence of the adaptive algorithm and usually is defined as the *step-size parameter*. Also,

$$\mathbf{C}_k(n) = \begin{pmatrix} c_k(0,\cdots,0;n) \\ \vdots \\ c_k(N-1,\cdots,N-1;n) \end{pmatrix}, \tag{6}$$

and the gradient vector $\boldsymbol{\Delta}_k(n)$ is defined as

$$\boldsymbol{\Delta}_k(n) = \begin{pmatrix} \frac{\partial e^2(n)}{\partial c_k(0,\cdots,0;n)} \\ \vdots \\ \frac{\partial e^2(n)}{\partial c_k(N-1,\cdots,N-1;n)} \end{pmatrix}. \tag{7}$$

It is taken into consideration that (*cf.* Equation (3))

$$\frac{\partial e^2(n)}{\partial c_k(i_1,\cdots,i_k;n)} = -2e(n)\frac{\partial z(n)}{\partial c_k(i_1,\cdots,i_k;n)}, \tag{8}$$

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where $\partial z(n)/\partial c_k(i_1, \cdots, i_k; n)$ can be written as (*cf.* Equation (1))

$$\frac{\partial z(n)}{\partial c_k(i_1, \cdots, i_k; n)} = \sum_{r=0}^{M-1} g(r; n) \frac{\partial y(n-r)}{\partial c_k(i_1, \cdots, i_k; n)}. \tag{9}$$

Here, $g(r; n)$ is given as

$$g(r; n) = \frac{\partial z(n)}{\partial y(n-r)} = h_1(r) + 2 \sum_{i=0}^{M-1} h_2(r, i) y(n-i) + 3 \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} h_3(r, i_1, i_2) y(n-i_1) y(n-i_2) + \cdots \tag{10}$$

Assuming that the μ_k chosen is sufficiently small, $\partial y(n-r)/\partial c_k(i_1, \cdots, i_k; n)$ can be approximated as (*cf.* Equation (2))

$$\frac{\partial y(n-r)}{\partial c_k(i_1, \cdots, i_k; n)} \approx \frac{\partial y(n-r)}{\partial c_k(i_1, \cdots, i_k; n-r)} = x(n-r-i_1) \cdots x(n-r-i_k). \tag{11}$$

Substituting Equations (9)–(11) in Equation (8), we have

$$\frac{\partial e^2(n)}{\partial c_k(i_1, \cdots, i_k; n)} = -2e(n) \sum_{r=0}^{M-1} g(r; n) x(n-r-i_1) \cdots x(n-r-i_k). \tag{12}$$

Remark 2

In Equation (10), it is assumed that the correct kernels of the nonlinear system $\boldsymbol{H}_{(q)}$ are known or have been estimated. The problem of estimating Volterra kernels for nonlinear systems is discussed, for example, in [8].

3. THE NONLINEAR FILTERED-X PREDICTION ERROR METHOD ALGORITHM

Prediction error methods (PEMs) are a family of parameter estimation methods that can be applied to a wide spectrum of model parameterizations. PEM has a close relationship with the maximum likelihood method . Therefore, it gives models with excellent asymptotic properties ([11, Chapter 7], [12, Section 4.4 and Chapter 5], [14, 15]).

The basic idea behind the prediction error approach is to describe the model as a predictor of the next output. Then, this predictor is parameterized in terms of a finite-dimensional parameter vector $\boldsymbol{\theta}$. Hence, a consistent estimate of $\boldsymbol{\theta}$ is determined from the model parameterization and the observed data set. In case the model has a different structure from the process, $\boldsymbol{\theta}$ is determined such that the prediction error is minimized under its structural constraints.

The Gauss–Newton algorithm [11, 12, 14] is a method used to solve nonlinear least squares problems. It can be seen as a modification of Newton's method for finding a minimum of a function. Unlike Newton's method, the Gauss–Newton algorithm has the advantage of the second derivatives of the cost function, which can be challenging to compute, not being required. The Gauss–Newton PEM algorithm has been used in [15, 16] for identification of nonlinear systems modeled using a Wiener model structure.

In this paper, the Gauss–Newton PEM algorithm is modified and applied on Figure 1 in order to estimate the predistorter coefficients. The modified algorithm, denoted as the Gauss–Newton NFx-PEM algorithm because of its similarity with the NFxLMS algorithm of Section 2, is derived by the minimization of the cost function [12]

$$V(\mathbf{C}) = \lim_{\mathcal{N} \to \infty} \frac{1}{\mathcal{N}} \sum_{n=1}^{\mathcal{N}} \mathrm{E}\left[e^2(n, \mathbf{C})\right], \tag{13}$$

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where $e(n, \mathbf{C})$ is the prediction error, which is defined as

$$e(n, \mathbf{C}) = d(n) - z(n, \mathbf{C}), \tag{14}$$

and $\mathbf{C}$ defined as

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_1(n) \\ \vdots \\ \mathbf{C}_p(n) \end{pmatrix}, \tag{15}$$

where $\mathbf{C}_k(n)$ is given by Equation (6).

The formulation of the NFxPEM algorithm requires the negative gradient of $e(n, \mathbf{C})$ with respect to $\mathbf{C}$, which is defined as

$$\boldsymbol{\psi} = -\frac{\mathrm{d}e(n, \mathbf{C})}{\mathrm{d}\mathbf{C}} = \begin{pmatrix} \boldsymbol{\psi}_1(n) \\ \vdots \\ \boldsymbol{\psi}_p(n) \end{pmatrix}, \tag{16}$$

where

$$\boldsymbol{\psi}_k(n) = \begin{pmatrix} \frac{\partial z(n)}{\partial c_k(0,\cdots,0;n)} \\ \vdots \\ \frac{\partial z(n)}{\partial c_k(N-1,\cdots,N-1;n)} \end{pmatrix}. \tag{17}$$

A straightforward analysis similar to Section 2 gives

$$\frac{\partial z(n)}{\partial c_k(i_1, \cdots, i_k; n)} = \sum_{r=0}^{M-1} g(r;n) x(n-r-i_1) \cdots x(n-r-i_k), \tag{18}$$

where $g(r;n)$ is given by Equation (10). Hence, the NFxPEM algorithm follows as (*cf.* [11, 12])

$$\begin{aligned} e(n, \mathbf{C}) &= d(n) - z(n, \mathbf{C}) \\ \lambda(n) &= \lambda_o \lambda(n-1) + 1 - \lambda_o \\ S(n) &= \boldsymbol{\psi}^{\mathrm{T}}(n)\mathbf{P}(n-1)\boldsymbol{\psi}(n) + \lambda(n) \\ \mathbf{P}(n) &= \left(\mathbf{P}(n-1) - \mathbf{P}(n-1)\boldsymbol{\psi}(n)S^{-1}(n)\boldsymbol{\psi}^{\mathrm{T}}(n)\mathbf{P}(n-1)\right)/\lambda(n) \\ \mathbf{C}(n) &= \mathbf{C}(n-1) + \mathbf{P}(n)\boldsymbol{\psi}(n)e(n, \mathbf{C}). \end{aligned} \tag{19}$$

Here, $\lambda(n)$ is a forgetting factor that grows exponentially to 1 as $n \to \infty$, where the rate λ_o and the initial value $\lambda(0)$ are design variables. The numerical values $\lambda_o = 0.99$ and $\lambda(0) = 0.95$ have proven to be useful in many applications [12]. Also, $\mathbf{P}(n) = n\mathbf{R}^{-1}(n)$, where $\mathbf{R}(n)$ is the Hessian approximation in the Gauss–Newton algorithm [11, 12]. The most common choice for the initial condition of $\mathbf{P}(n)$ is $\mathbf{P}(0) = \rho\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and ρ is a constant that reflects our trust in the initial parameter vector $\mathbf{C}(0)$. In case of no prior knowledge, $\mathbf{C}(0) = \mathbf{0}$, and ρ is large to speed up convergence to the true parameter vector.

4. COMPUTATION COMPLEXITY OF THE NONLINEAR FILTERED-X PREDICTION ERROR METHOD ALGORITHM

The CC of the NFxPEM algorithm (19) is higher than that of the NFxLMS algorithm because of the fact that the NFxPEM algorithm requires the recursive computation of the matrix $\mathbf{P}(n)$ in addition to the parameter vector $\mathbf{C}(n)$. See [17] for a detailed discussion on the computational complexities of these algorithms. In this section, the CC per sample or iteration is given by comparing the NFxPEM and NFxLMS algorithms in the case of predistortion of Volterra systems. As in Section 2, for a Volterra system $\boldsymbol{H}$ of order q with memory length M, the number of the parameters

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is $\sum_{i=1}^{q} M^i$. Also, the predistorter $\boldsymbol{C}$ is chosen as a pth-order Volterra system with memory N and $\sum_{i=1}^{p} N^i$ parameters. The approximated addition per sample (+/sample) and multiplication per sample (×/sample) for these two online adaptation algorithms are given in Table I. Note that here we assume that the nonlinear physical system has been identified; otherwise, an extra CC should be added for identifying the Volterra system.

For us to have a direct feeling of the CC comparison, let us consider the predistortion of a second-order Volterra system and assume the following: $p = q = 2$ and $M = N = 4$. We can conclude that the CC of the adaptation algorithms in Table II and the overall CC represent the total additions and multiplications required for the convergence of the adaptation algorithm. Although the CC per iteration of the NFxLMS algorithm is lower than that of the NFxPEM algorithm, the overall CC of the NFxLMS algorithm is higher because of its slow convergence.

5. SIMULATION STUDY

In this section, a comparative simulation study between the NFxLMS algorithm and the NFxPEM algorithm is given. In Example 1, the nonlinear system is assumed to be a second-order Volterra system. In Example 2, the simulated platform of the digital subscriber line (DSL) system—provided by Infineon Technologies—is considered, taking into account all elements in the transmitter and receiver paths. The nonlinear system in this case is the model of the line driver (LD), which is selected as a fifth-order Volterra system. See [16–20] for more details on modeling the LD in DSL systems. In these simulations, it is assumed that $\tilde{\boldsymbol{H}}_{(q)} = \boldsymbol{H}_{(q)}$ (*cf.* Remark 1 and Figure 1).

Example 1

The nonlinear system $\boldsymbol{H}_{(q)}$ is a known second-order Volterra system. The adaptive predistorter $\boldsymbol{C}_{(p)}$ is also assumed to be a second-order Volterra filter. This means that $q = p = 2$. Also, the number of memories in the adaptive Volterra predistorter is chosen as $N = 3$. The input–output relation of the nonlinear system $\boldsymbol{H}_{(2)}$ is chosen to be

$$z(n) = \boldsymbol{H}_{(2)}[y(n)] = \boldsymbol{H}_1[y(n)] + \boldsymbol{H}_2[(y(n)], \tag{20}$$

Table I. Computation complexity per sample of the NFxLMS and NFxPEM algorithms for predistortion of qth-order Volterra systems.

Algorithm	+/sample	×/sample
NFxLMS	$M\sum_{i=1}^{q-1} M^i + (M+1)\sum_{i=1}^{p} N^i$	$(M+1)\sum_{i=1}^{p}[i \times M^i] + M\sum_{i=1}^{q-1}[i \times M^i]$ $+3\sum_{i=1}^{p} N^i$
NFxPEM	$(M+2)\sum_{i=1}^{p} N^i + 5(\sum_{i=1}^{p} N^i)^2$ $+M\sum_{i=1}^{q-1} M^i$	$6(\sum_{i=1}^{p} N^i)^2 + (M+1)\sum_{i=1}^{p}[i \times N^i]$ $+M\sum_{i=1}^{q-1}[i \times M^i]$

NFxLMS, nonlinear filtered-x least mean squares; NFxPEM, nonlinear filtered-x prediction error method.

Table II. CC of the NFxLMS and NFxPEM algorithms for predistortion of a second-order Volterra system.

Algorithm	+/sample	×/sample	Convergence (samples)	Overall CC
NFxLMS	180	384	$>1.2 \times 10^4$	$>6.78 \times 10^6$
NFxPEM	2200	2724	800	3.94×10^6

CC, computation complexity; NFxLMS, nonlinear filtered-x least mean squares; NFxPEM, nonlinear filtered-x prediction error method.

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where the first-order kernel vector $\boldsymbol{H}_1$ is given as

$$\boldsymbol{H}_1 = \begin{pmatrix} 0.5625 & 0.4810 & 0.1124 & -0.1669 \end{pmatrix} \tag{21}$$

and the second-order kernel matrix $\boldsymbol{H}_2$ is

$$\boldsymbol{H}_2 = \begin{pmatrix} 0.0175 & 0 & 0 \\ 0 & 0 & -0.0088 \\ 0 & -0.0088 & 0 \end{pmatrix}. \tag{22}$$

The input signal to the predistorter is chosen as a random signal with uniform distribution over $(-1, 1)$ with data length 2×10^4, and the frequency band is limited as performed in [3] to prevent aliasing. Because the nonlinear systems that are considered in this study are minimum-phase systems, the time delay τ in Equation (4) is set equal to 0 (*cf.* Remark 1). Hence, the desired signal $d(n)$ is chosen to be equal to the input signal $x(n)$ in an additive white Gaussian noise such that a signal-to-noise ratio of 40 dB is achieved.

As a measure of performance, the mean square distortion (MSD) of the system consisting of the predistorter plus the nonlinear system is defined as

$$MSD = 10 \, \log_{10} \left(\frac{\widehat{\mathrm{E}}\{e^2(n)\}}{\widehat{\mathrm{E}}\{d^2(n)\}} \right), \tag{23}$$

where $\widehat{\mathrm{E}}\{.\}$ is the mean obtained by 10^3 independent experiments.

The MSD comparison between the NFxLMS and NFxPEM algorithms is given in Figure 2. The step size of the NFxLMS algorithm is $\mu = 0.1$, and the matrix is $\mathbf{P}(0) = 10\mathbf{I}$ for the NFxPEM algorithm. The distortion of the nonlinear system without predistorter was -16.04 dB. The NFx-PEM algorithm gives a lower distortion than the NFxLMS algorithm. On average, the NFxPEM algorithm achieves about -39.69 dB, and the NFxLMS algorithm achieves around -31.93 dB. On the other hand, the NFxPEM algorithm converges much faster than the NFxLMS algorithm.

Figure 3 shows power spectral densities of the output signals of the nonlinear system with and without predistorter. From this figure, we can see that as compared with using the NFxLMS algorithm, the predistorter using the NFxPEM algorithm can reduce the spectral regrowth more effectively.

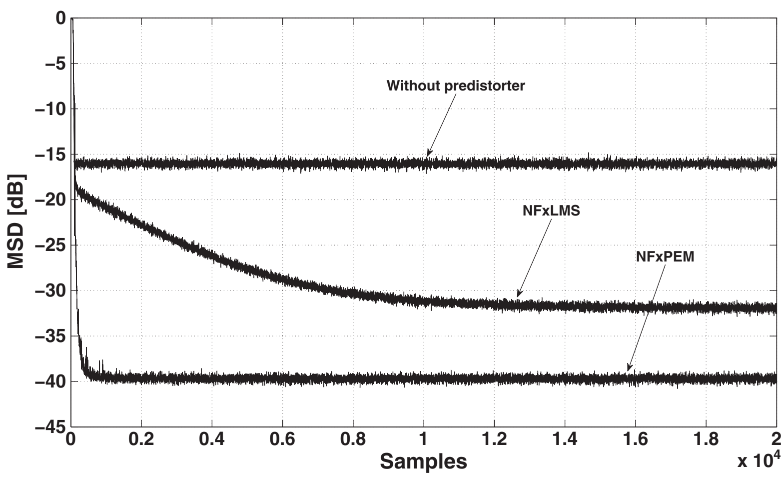

Figure 2. Distortion comparison between the nonlinear filtered-x least mean squares (NFxLMS) and nonlinear filtered-x prediction error method (NFxPEM) algorithms. MSD, mean square distortion.

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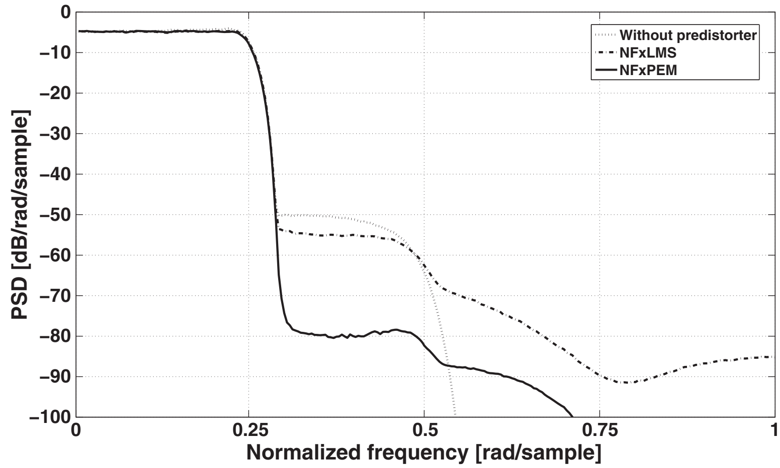

Figure 3. Power spectral densities (PSDs) for *signal-to-noise ratio* = 40 dB. NFxLMS, nonlinear filtered-x least mean squares; NFxPEM, nonlinear filtered-x prediction error method.

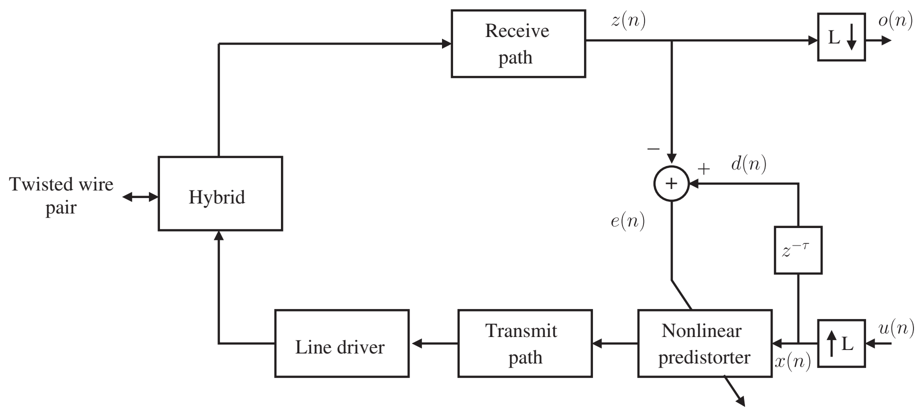

Figure 4. Block diagram of the application of nonlinear predistortion xDSL systems.

Example 2

The block diagram in Figure (4) shows an application of nonlinear predistortion in a DSL simulation platform provided by Infineon Technologies . A nonlinear predistorter is used to precompensate the LD circuit, which is the main source of nonlinear distortion. During the start-up phase of the DSL system, a predetermined discrete multitone (DMT) signal can be sent as a training sequence in order to estimate the coefficients of the nonlinear predistorter.

In this simulation, a real LD model for the next-generation DSL systems is considered. It is a fifth-order Volterra system. The memory lengths of the first-order to fifth-order kernels are 15, 0, 5, 0, and 2, respectively. The total number of nonzero parameters is 50. The nonlinear predistorter is also assumed to be a fifth-order Volterra system with the same memory lengths. The training sequence $u(n)$ is a DMT signal, which is defined as

$$u(n) = \sum_{k=0}^{K} 2|U_k| exp\left[j \left(2\pi \frac{f_{\max}}{K} kn + \varphi_k \right) \right], \tag{24}$$

where U_k are user-defined amplitudes and φ_k are random phases with uniform distribution and $\mathrm{E}\{e^{j\varphi_k}\} = 0$. The number of tones is $K = 64$ and $f_{\max} = 4312.5$ Hz. The data length is 2^{13}

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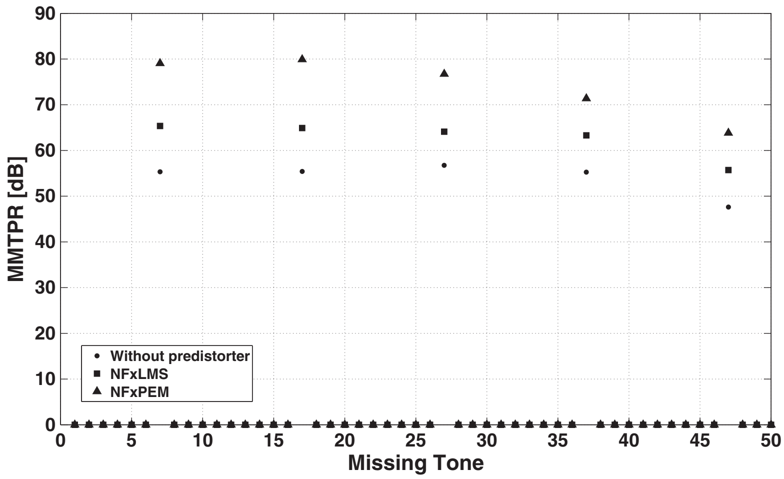

Figure 5. Mean multitone power ratio (MMTPR) values. NFxLMS, nonlinear filtered-x least mean squares; NFxPEM, nonlinear filtered-x prediction error method.

samples. Here, the random phases generate training sequences with different properties in each of the experiments, that is, the crest factor and root mean square value. In this simulation, the crest factor and root mean square value of our training sequences are regulated to 1 and 0.5, respectively. The upsampling and downsampling factor is $L = 5$. The transmit and receive paths are simply modeled as low-pass filters using eighth-order Butterworth filters with a normalized corner frequency of 0.5π because they are not the main concern of this study (see [17] for more details).

The step size of the NFxLMS algorithm μ is chosen as 1×10^{-4} because a larger step-size value could cause instability of the simulation. The matrix $\mathbf{P}(0)$ for the NFxPEM algorithm is chosen as $1 \times 10^{-3}\mathbf{I}$.

The mean multitone power ratio (MMTPR) [21] is used to evaluate performance. The MMTPR for each tone of the output signal $o(n)$ is defined as

$$MMTPR_k = \frac{\widehat{\mathrm{E}}\{T_k\}}{\widehat{\mathrm{E}}\{S_k + \sum_{j,j \neq k} I_j\}}, \quad k = 1, 2, \cdots, K, \tag{25}$$

where $\widehat{\mathrm{E}}\{.\}$ is the mean obtained by 100 independent experiments, T_k and S_k stand for the transmitted and noise powers of the kth tone, respectively, and $\sum_{j,j \neq k} I_j$ is the inter-modulation power of the kth tone from the other $K - 1$ tones.

Remark 3

In the start-up phase (off-line case) of the DSL system, the DMT signal is used as an input, and the output $o(n)$ is measured and used to generate the error signal required for the design of the algorithms. Here, we do not measure the analog output of the LD in order to avoid using an extra analog to digital converter.

Remark 4

The basic idea used to evaluate the MMTPR is to transmit the DMT signal by missing or removing the kth tone and measure the received power at this tone frequency. Because of the nonlinear effect, this measured power represents the inter-modulation and noise power, that is, $S_k + \sum_{j,j \neq k} I_j$.

Figure 5 shows the MMTPR values of the system output with and without predistorter. The MMTPR values of the tones with index $\{k | k = 7, 17, 27, 37, 47\}$ were measured. From this figure, we can see that the predistorter using the NFxPEM algorithm can achieve much better MMTPR

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values as compared with the NFxLMS algorithm and hence more effectively compensates the nonlinear distortion than the NFxLMS algorithm for DSL systems.

6. CONCLUSIONS

Predistortion of nonlinear systems using a nonlinear Volterra predistorter is considered in this paper. A novel NFxPEM algorithm for estimating the kernels of the predistorter has been introduced. The NFxPEM algorithm is compared with the NFxLMS algorithm using numerical simulations and application on the DSL simulation platform developed by Infineon Technologies. The simulation results show that the new NFxPEM algorithm achieves much higher convergence speed, more significant reduction in nonlinear distortion, and more suppression of spectral regrowth as compared with the NFxLMS algorithm.

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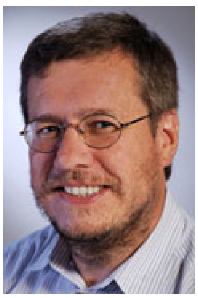

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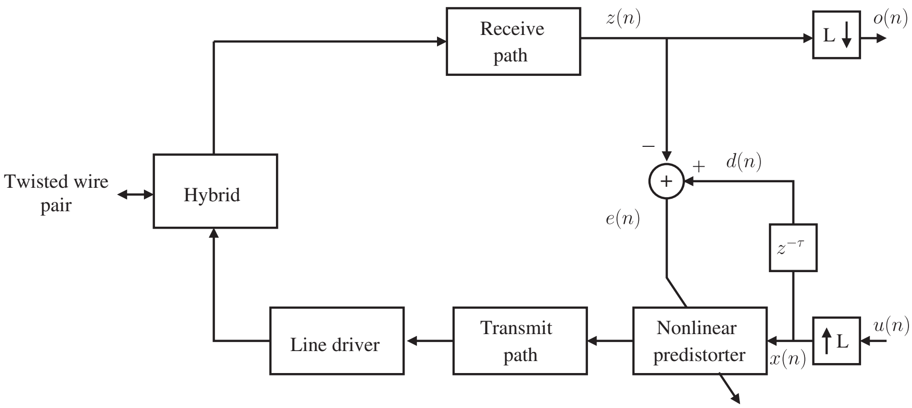
Research Article

A nonlinear filtered-x prediction error method algorithm for digital predistortion in digital subscriber line systems

Emad Abd-Elrady, Li Gan and Gernot Kubin



In this paper, the prediction error method (PEM) is used to derive a novel nonlinear filtered-x prediction error method (NFxPEM) algorithm. A simulation study on Volterra systems shows that the NFxPEM algorithm more significantly suppress spectral regrowth and converges much faster than the NFxLMS algorithm. Also, the NFxPEM algorithm is used in this paper to design more efficient digital predistorter as compared to the NFxLMS algorithm for digital subscriber line systems.