

# Development of a Dual Frequency Software-based GNSS Receiver

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## ABSTRACT

Throughout the last decade, a trend towards the development of software-based Global Navigation Satellite System (GNSS) receivers has evolved in the fields of scientific research. Software-based GNSS receivers feature a high flexibility for the adaptation to several applications, serve as a development platform for studying new algorithms and techniques, and especially have the advantage of requiring only few hardware parts. The main focus of current developments is the single frequency approach, aiming at mass market applications. The resulting position accuracy is insufficient for many applications, e.g., for the automotive domain. Modern driver assistance systems require position accuracies in the range of one lane width and even better (< 1m). Also safety relevant applications pose strong demands on higher position accuracy. Beside the errors introduced by

the satellites (satellite orbit, satellite clock), the troposphere, and the receiver clock, the ionosphere is one of the biggest error sources, when neglecting local effects like multipath for the moment. Adding a second measurement of a signal of a different carrier frequency, the error due to the ionosphere can be eliminated. Furthermore the integrity can be increased, which is essential for a wide range of applications.

The current project work aims at developing a dual frequency software-based GNSS receiver to fulfil the above mentioned accuracy requirements. In its present implementation the receiver is capable of GPS code measurements making use of both available civil signals (L1 C/A and L2C). Efforts in modernizing the GPS satellites are still ongoing, but there are already (as at August 2010) eight Block IIR-M satellites and one IIF satellite in orbit, transmitting the second civil signal L2C on the L2 carrier. By using this second frequency it is possible to reduce/eliminate the ionospheric effect.

After a brief introduction on the overall concept, the paper mainly focuses onto the implementation of the dual frequency software part. Afterwards, the main components of the receiver, i.e., the Digital Signal Processing (DSP) module, the Position, Velocity, and Time (PVT) module, and the acquisition aiding module are discussed in more detail. Concluding, some results of testing and validating the software-based receiver are presented. Thereby, the special emphasis is placed on the comparison between the single frequency and the dual frequency approach.

## INTRODUCTION

The Global Navigation Satellite System (GNSS) user receiver represents a key technology and is the central element of the user segment. It is the main physical interface between the system and the user and transforms the GNSS signal-in-space into services for the citizen. During the last years, a significant trend towards the development of software-based GNSS receivers has evolved. Within this movement, following [1], a new trend in the GNSS receiver design process, the software defined radio (SDR) approach, has been developed. Within this approach the digitization moves closer to the receiver's antenna, compared to FPGA or ASIC implementations. Thus, SDR approaches make use of higher frequencies and wider bandwidth. This is ideal for designing multi-system (e.g., GPS + Galileo) and multi-frequency GNSS receivers. The main focus of current developments is the single frequency approach. However, the resulting position accuracy is not sufficient for many applications, especially for the automotive domain. Modern driver assistance systems need position accuracies in the range of one lane width and even better ( $\leq 1\text{m}$ ). Nowadays, also security relevant applications and related applications have a strong demand on higher position accuracy. Beside the errors introduced by the satellites (satellite orbit, satellite clock), the troposphere, and the receiver clock, the ionosphere is one of the biggest error sources. Adding a second measurement,

using a different carrier frequency, the error due to the ionosphere can be eliminated. Furthermore, the integrity can be increased, and this is inevitable for a wide range of applications, especially for the security relevant ones.

To fulfil the above mentioned accuracy requirements, within the project "SoftGNSS 2 – a dual frequency software-based GNSS receiver", the design and implementation of a dual frequency software-based GNSS receiver is accomplished. SoftGNSS 2 is a 16 months project, which started in July 2009, and is managed by the Austrian Research Promotion Agency (FFG) through funds of the Federal Ministry for Transport, Innovation and Technology (BMVIT). The project consortium consists of three partners: TeleConsult Austria GmbH, Signal Processing and Speech Communication Laboratory of Graz University of Technology, and the Institute of Navigation and Satellite Geodesy at Graz University of Technology. Currently, the SoftGNSS 2 project is in its final phase. All developments and tests have been almost completed.

The main objective of this project was to implement a dual frequency software-based GNSS receiver, first focussing on GPS L1/L2, or to be more specific C/A and L2C pseudorange measurements, with the aim at a future adaptation to Galileo and other GNSS.

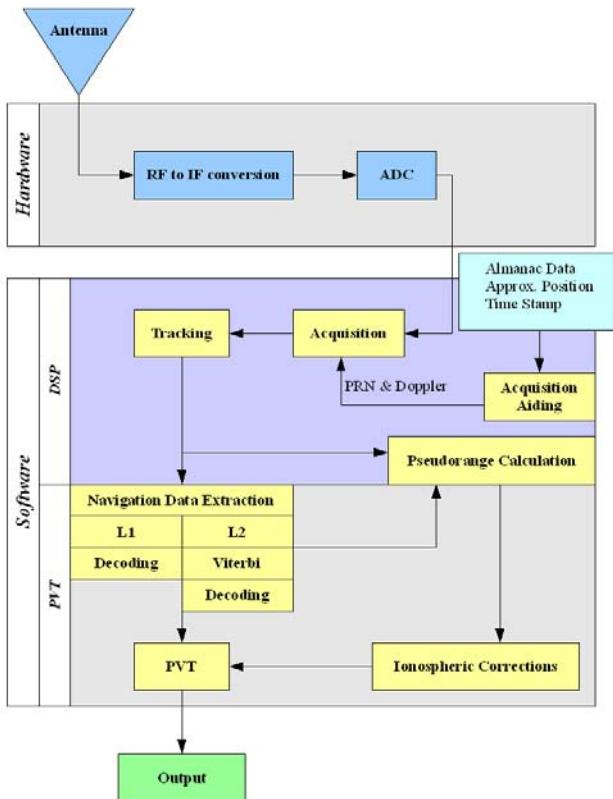
In a first step, as a proof of concept, the receiver is only capable of GPS code measurements but makes use of both current civilian signals. The second civilian signal L2C, carried by the second carrier frequency L2, is at the moment under construction, but already 9 satellites (Block IIR-M and IIF satellites, as at August 2010) transmitting this signal, are in orbit.

The L2C signal is not just a redundant copy of the L1 C/A signal with a different carrier frequency. The advantage of using L2C is the better performance of the overall signal structure. Due to the time-multiplexed nature of L2C (cf. [2]), different correlators have to be used for tracking, and thus, new digital signal processing chains are required. Using a second measurement with a different frequency, the ionospheric effects can be eliminated and hence, the position accuracy improves. When using the ionosphere free linear combination, special emphasis must be put on avoiding the increase of the measurement noise.

## SYSTEM ARCHITECTURE

The system architecture of the software-based receiver can be split up into three main parts. The first part covers the necessary hardware which is responsible for the radio frequency (RF) to digital conversion. The second part of the architecture is the Digital Signal Processing (DSP) module, which performs acquisition and tracking of the signal, as well as data preparation tasks. Finally the Position, Velocity, and Time (PVT) module contains all necessary algorithms for the computation of the receiver position out of the raw observables. The current software implementation comprises both MATLAB and C++ modules. The DSP is implemented in MATLAB to obtain the necessary flexibility during the research of signal processing algorithms, while the PVT is implemented in

C++. Both modules are connected via the MATLAB Executable file (MEX) interface.



**Figure 1 Overall SoftGNSS 2 architecture**

The system architecture, as described before, is shown in Figure 1. The first part, as already mentioned, covers the necessary hardware, the so-called RF front-end, which is responsible for the RF to digital conversion. Due to the fact that a RF front-end design is a very time and cost consuming process, the consortium pursued the strategy to buy off-the-shelf components to cover this task. The DSP module contains several sub modules for the acquisition, the tracking, and the raw data preparation. It has to interact tightly with the third part - the Position, Velocity and Time (PVT) module. This module contains all necessary algorithms for the computation of the user's position out of the raw observables and broadcast ephemeris. Additionally, the navigation data decoding is implemented in this module. Further, an ionospheric correction module is implemented to advance the receiver's position accuracy. The output of the receiver can be either user-defined binary messages or NMEA messages.

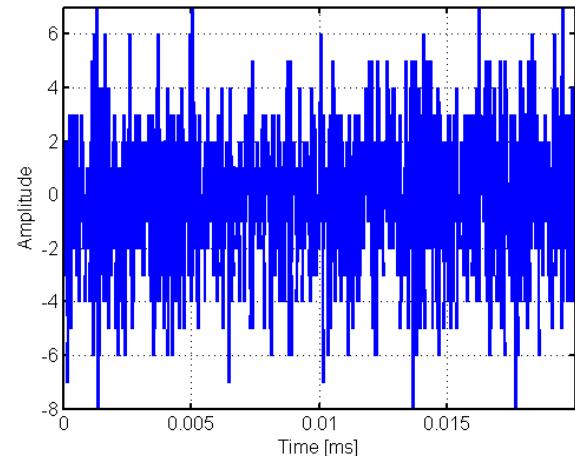
#### Radio Frequency Front-end

The GNSS antenna, as well as, the RF front-end represent the link between the GNSS signal in space and the software part (cf. [9]). As already mentioned the design of a front-end is very expensive and time consuming. Thus, after thorough investigations an off-the-shelf front-end was used within the project. Among different front-end suppliers, Fraunhofer IIS (Institute for Integrated Circuits,

Nuremberg, Germany) offers a multiband front end (cf. [3]), which is capable of receiving all presently known and future GNSS-signals within the L1, L2, and L5 frequency band. The important parameters for the signal processing are:

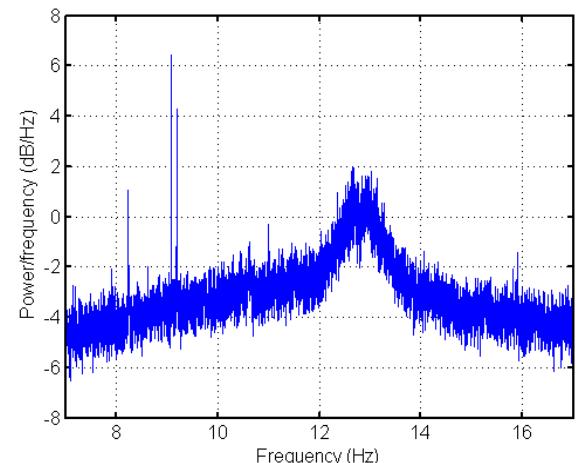
- centre frequency: 1575,42 MHz, 1227,60 MHz, and 1176,45 MHz,
- bandwidth: 18 MHz,
- intermediate frequency: 12,8 MHz,
- sampling rate: 40,96 MHz, and
- two or four bit sample resolution.

The digitized data are available through two USB ports in real-time or can be stored in a binary file for post-processing tasks.



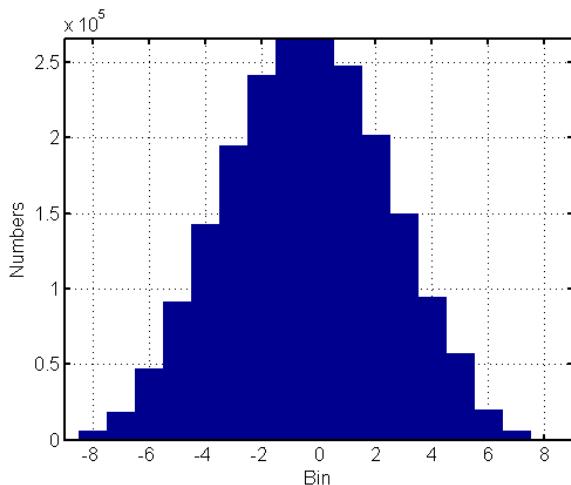
**Figure 2 Time domain plot of digital IF signal**

Figure 2 provides the time domain representation of 200 ms digital IF signal, whereas Figure 3 shows the power spectral density of the same signal part.



**Figure 3 Frequency spectrum of digital IF signal**

Figure 4 shows a histogram of the input signal. The 16 different levels proof that in this case a four bit analogue-to-digital conversion was used.

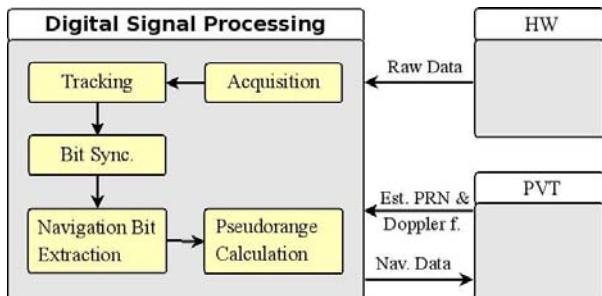


**Figure 4 Four bit quantization of the digital IF signal**

During the development, a GNSS constellation and performance simulator (GIPSIE®) (cf. [4]) developed by TeleConsult Austria GmbH was used as an additional data source. The simulator provided highly reproducible scenarios during the software testing. For testing purposes the same signal parameters, compared to the front end, were used to generate the simulator data.

#### Digital Signal Processing module

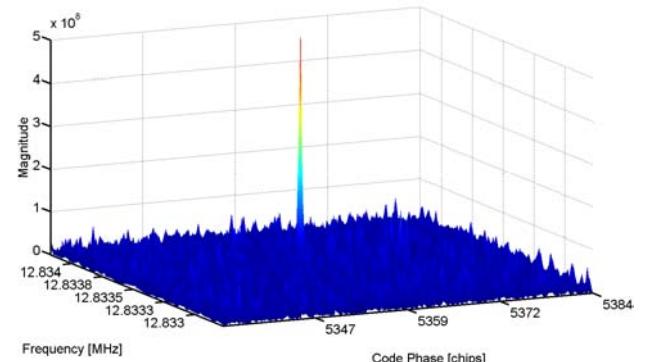
After the analogue-to-digital conversion, the digital IF signal is fed into the DSP module. The DSP module has three main stages. In the first stage, all visible satellites are determined and their characteristic parameters, such as Doppler shift of the carrier and the actual code phase of the PRN code, are measured coarsely. Accordingly, this first stage is called acquisition. In the subsequent tracking stage, these parameters are refined and tracked by control loops (Costas loop and delay-locked loop). In the last stage, the fact that the satellites are in track is exploited to extract the navigation data modulated on the PRN code and to perform pseudo-range measurements. A schematic view of this process is outlined in Figure 5.



**Figure 5 Digital Signal Processing Block**

To speed up the acquisition, an acquisition aiding module is implemented. This module calculates out of almanac data, a rough user position and time information, all possible visible satellites and corresponding start values for the Doppler and initial code phase. Thus, the search domain and the number of computation steps within the

acquisition phase can be reduced significantly. If one of the needed input sources is missing, the algorithm still provides a priority list of possible visible satellites. Also in this case an improvement regarding the acquisition time can be shown. Figure 6 shows the 2D acquisition result of a present satellite signal using the L2C signal.



**Figure 6 Acquisition result for L2C PRN 12**

In a combined L1/L2C acquisition scheme, the acquisition of L1 is performed first where the PRNs are screened for in the order given by the priority list of visible satellites. For each PRN, the expected Doppler frequency that is provided by the acquisition aiding module is used as an initial value for the Doppler frequency search which screens the vicinity of the expected frequency gradually. To accelerate this stage of the acquisition, a search over multiple code phases is performed simultaneously (parallel code phase search, [5]) for each investigated Doppler frequency. Moreover, the computational complexity of the demodulation is reduced by the shifting replica approach proposed in [6]. The obtained L1 acquisition results are used in turn to perform aided acquisition for L2C. The PRNs of the acquired satellites are employed to update the priority list processed by the L2C acquisition stage. Additionally, the acquired L1 Doppler frequency can be used to compute Doppler estimates for L2C [7], which serve as initial values for the L2C frequency search. This is especially beneficial as the number of frequency bins to be searched for L2C increases in accordance with the integration time [8].

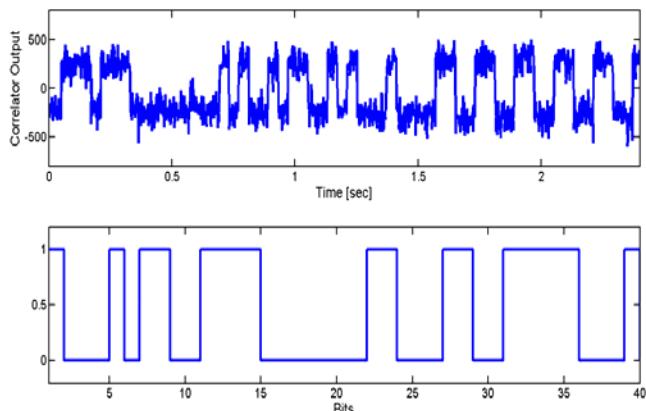
#### Position, Velocity, and Time module

Finally, the DSP module continuously outputs the navigation bits (prompt correlator in phase) in case of the L1 frequency to the PVT module. The PVT module awaits the necessary amount of navigation bits and checks for the occurrence of the preamble (cf. [2]). In case of a valid preamble, the parity check is done and the navigation message is decoded. As a feedback to the DSP module the exact occurrence of the preamble is returned in combination with the decoded time information. With this information it is now possible for the DSP module to compute the raw pseudoranges and to provide them to the PVT module. The actual position computation starts when the necessary subframe data, in case of L1 - subframe 1,

2, 3, and 4 - are successfully decoded and if at least four pseudorange measurements are available.

In case of L2C the DSP module forwards the output of the prompt correlator in phase to the PVT module. According to [2], the signal configuration of L2C can either contain the NAV data at 50 bps, or the NAV data at 25 bps with FEC encoding, or the CNAV data at 15 bps with FEC encoding. Since there is no flag within the data train which indicates the actual configuration, the software has to try – especially during the initial phase of L2C – every option to succeed.

If the navigation message on L2C is encoded using a forward error correction (FEC) algorithm (cf. [2]), a Viterbi decoder is used, to decode the navigation bits first. In case of the SoftGNSS 2 project, a soft-decision decoder is used to decode the navigation bits. Tests on the 13<sup>th</sup> of August 2010 at 14:30 in Graz showed that at this time the CNAV dummy message was broadcasted. Figure 7 shows the correlator input, in fact the up-sampled navigation chips (cf. [5]) - in the upper plot. The lower plot shows the first 40 bits of the decoded CNAV dummy message.



**Figure 7 Navigation message decoding in case of L2C measurements**

After decoding the subframe data, the actual position computation takes place. The PVT module is capable of using either a least squares adjustment or a Kalman filter to compute the final user position. Within the single frequency mode a tropospheric and the broadcast ionospheric correction model is used to account for the propagation effects. Methods for modelling the tropospheric and ionospheric effects are for example described in [9] or [10].

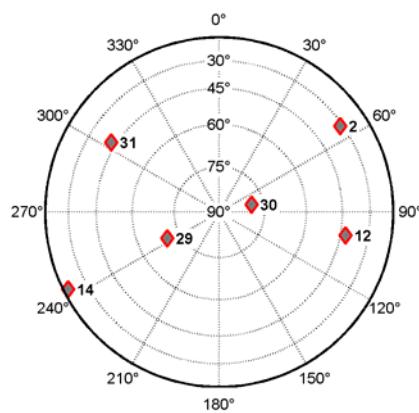
Following [9], the most efficient method of eliminating the ionospheric refraction is by using two signals with different carrier frequencies. Thus, an ionosphere-free linear combination  $R_c$  out of the code pseudorange measurements from both frequencies ( $R_1$  and  $R_2$ ) can be formed:

$$R_c = \left[ R_1 - \frac{f_2^2}{f_1^2} R_2 \right] \left( \frac{f_1^2}{f_1^2 - f_2^2} \right) = (\rho + c\Delta\delta)$$

**Equation 1 Ionosphere-free linear combination**

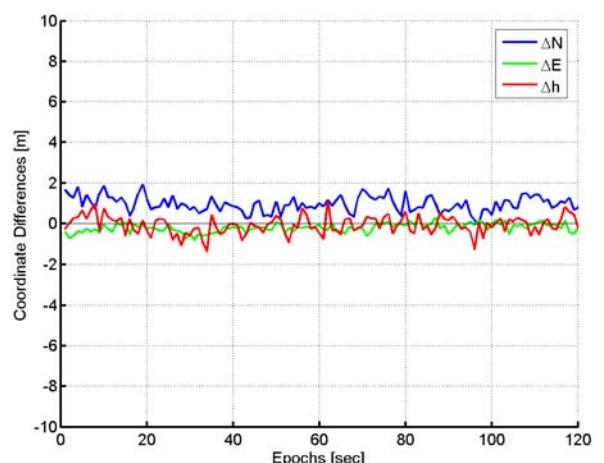
Thereby  $f_1$  and  $f_2$  denote the frequencies of the respective carrier. The ionosphere-free linear combination has one drawback – the amplification of the measurement noise due to the linear combination of the measurements. When applying variance propagation to Equation 1, it leads to a factor of about 3 for the amplification of noise.

In the following, some results of a test data set, collected on the 13<sup>th</sup> of August 2010 at 14:30 in Graz are presented. The antenna was set-up on an observation pillar with known coordinates. Besides using the multiband front end, a Javad Sigma multi frequency receiver, capable of receiving L1, L2, and L5 signals, was used for verification purposes. The actual GPS constellation, when using an elevation mask of 10°, is presented Figure 8.



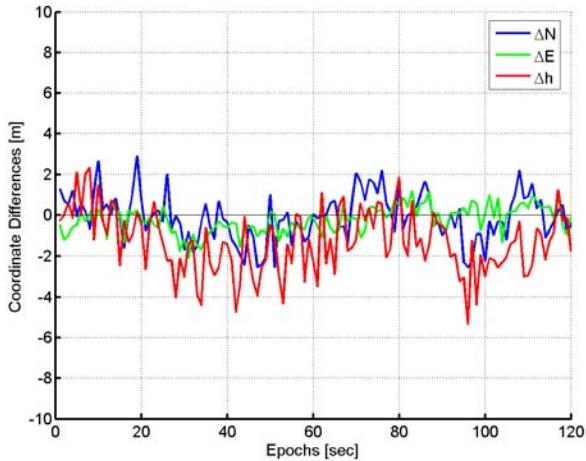
**Figure 8 Sky plot for Graz at the 13<sup>th</sup> of August 2010**

The gathered raw data were processed using the SoftGNSS 2 receiver. Figure 9 shows the UTM coordinate differences in meters in North, East, and height component between the single frequency solution and the reference coordinates. It must be mentioned that a Hopfield model was used to compensate for the tropospheric effect and the broadcasted Klobuchar parameters were used for compensating the ionospheric effect (cf. [11]).



**Figure 9 Coordinate differences in meters  $\Delta N$  (blue),  $\Delta E$  (green), and  $\Delta h$  (red) for the single frequency solution**

Due to the fact, that only three out of six satellites (PRN 31, 12, and 29) provided L2C pseudorange measurements, P2 measurements, taken by the Javad receiver, were used instead. The advantage of the ionosphere-free combination is the elimination (or more precisely, the reduction) of the ionospheric effect (cf. [9]). The before mentioned amplification of noise due to the formation of the linear combination is visible in Figure 10.



**Figure 10 Coordinate differences in meters  $\Delta N$  (blue),  $\Delta E$  (green), and  $\Delta h$  (red) for the dual frequency solution**

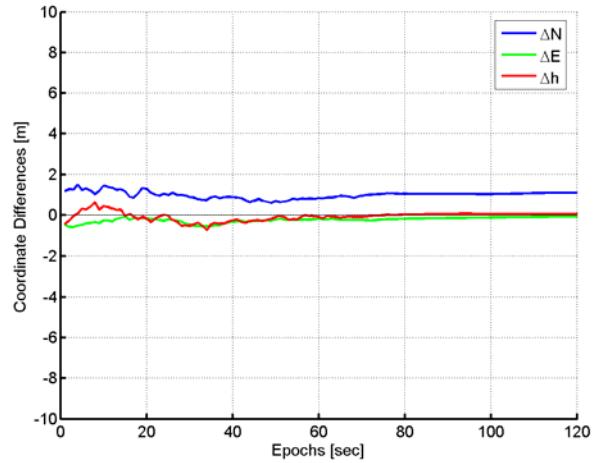
Thus, it is inevitable to put more effort into the computation of the linear combination. [10] states that the ionospheric effect can be averaged over time. Thus, a carrier smoothing is applied to the linear combination. In [9] and [12] code pseudorange smoothing using a time-dependent weight factor (Hatch filter) is described. The smoothed code pseudorange for one satellite is obtained by:

$$R(t_i)_{sm} = \omega \cdot R(t_i) + (1 - \omega) \cdot (R(t_{i-1})_{sm} + (\Phi(t_i) - \Phi(t_{i-1})))$$

**Equation 2**

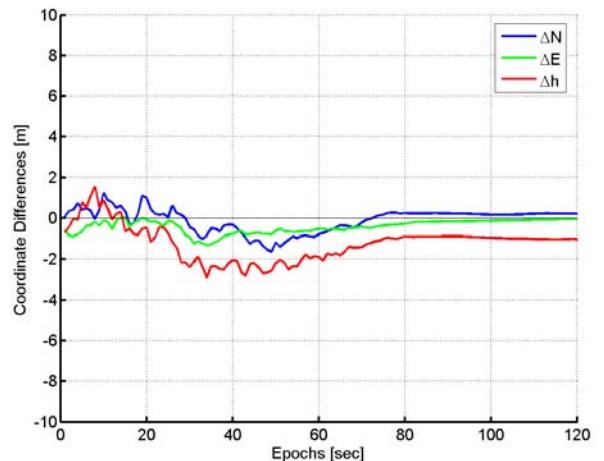
Where  $R(t_i)$  and  $\Phi(t_i)$  denote the code pseudorange and phase measurement, and  $\omega$  represents the weight factor.  $R(t_{i-1})_{sm}$  represents the smoothed pseudorange of the previous epoch. For the first epoch  $i=1$  the weight factor is set to one. In this case the code pseudorange has the full weight. For the consecutive epochs, the weight of the code pseudorange is continuously reduced and the weight of the phase measurements is increased. In case of cycle slips the algorithm will fail. Thus, a check on cycle slips or even cycle slip repair strategies have to be implemented.

When applying this weighted smoothing algorithm to the single frequency algorithm and looking at the coordinate differences (cf. Figure 11) the smoothing effect is visible.



**Figure 11 Coordinate differences in meters  $\Delta N$  (blue),  $\Delta E$  (green), and  $\Delta h$  (red) for the single frequency solution with applied carrier smoothing**

When combining this algorithm and the dual frequency algorithm, using the linear combination from Equation 1, the coordinate differences with respect to the reference coordinates (cf. Figure 12) show smoother characteristics than before.



**Figure 12 Coordinate differences in meters  $\Delta N$  (blue),  $\Delta E$  (green), and  $\Delta h$  (red) for the dual frequency solution with applied carrier smoothing without DCBs**

Comparing the solutions, no obvious benefit of the dual frequency algorithm is visible. Within [2] the topic of inter-signal group delay differential correction is discussed. The civil dual frequency user shall correct for the group delay and ionospheric effects by applying a modified linear combination using inter-signal correction (ISC) parameters.

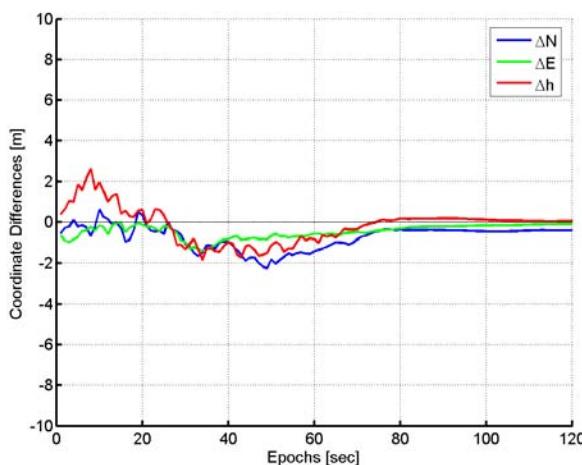
$$R_c =$$

$$\left[ R_1 + c \cdot ISC_1 - \frac{f_2^2}{f_1^2} (R_2 + c \cdot ISC_2) \right] \left( \frac{f_1^2}{f_1^2 - f_2^2} \right) - c \cdot T_{GD}$$

### Equation 3 Modified ionosphere-free linear combination

The ISC parameters will be available through the CNAV message as message type 30 data. As mentioned above, currently only message type 0 (dummy message) is broadcasted and thus no ISC parameters are available. Additional information regarding the need and the usage of inter-signal corrections can be found in [13].

Instead of using the ISC parameters, which were not available, differential code biases (DCB), obtained from CODE (Centre for orbit determination in Europe) (cf. [14]) were used for the next computation step.



**Figure 13 Coordinate differences in meters  $\Delta N$  (blue),  $\Delta E$  (green), and  $\Delta h$  (red) for the dual frequency solution with applied carrier smoothing**

The results of the dual frequency algorithm using DCB values are shown in Figure 13. The coordinate differences are below one meter and no fluctuations are visible after 80 epochs.

Beside the mentioned algorithms a weighting algorithm is used to properly weight the different observations. Thereby, beside an elevation dependent model, also information from the smoothing algorithm is used. Additionally, out of the dual frequency measurements the ionospheric effect is calculated and compared to the model parameters. This relation is then used in the weighting algorithm as well. Thus, it is possible to apply a proper weighting of dual frequency observations with respect to single frequency observations.

### Conclusion and Outlook

Within the SoftGNSS 2 project, a dual frequency software-based GNSS receiver, capable of processing the

civil C/A and the civil L2C signal, was developed. The paper presented the architectural design with special emphasis on the dual frequency part.

For testing and evaluating the performance of the developed receiver, several tests were accomplished. In first step, the software-based receiver was tested using the GIPSY® constellation and performance simulator. In a next step, real data were collected using the multiband front-end.

First results of using dual frequency observations were presented and the need and benefit of carrier smoothing was pointed out. Also the problem of currently missing inter-signal correction parameters was discussed.

In the following months, even more satellites capable of transmitting L2C will be launched. Thus, it soon will be possible to have the necessary amount of at least four satellites transmitting L2C signal at one and the same time. Additional effort will be put on the topic of the ISC parameters and their impact on the dual frequency solution.

Currently the receiver works in the post-processing mode, but in the future it is envisaged to reach real-time capability.

Further, the adaptation to other GNSS, especially Galileo, is planned and thus, the availability and the position accuracy will be further increased. First tests regarding the acquisition of GIOVE-A and GIOVE-B look very promising.

### ACKNOWLEDGMENTS

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