Antenna Diversity for OFDM using Cyclic Delays

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***Abstract*--A new technique is presented for exploiting antenna diversity in OFDM systems, which can be applied at the transmitter and/or at the receiver. The core idea concerns the introduction of cyclic delays to the effective parts (FFT-parts) of the OFDM signals transmitted/received via several antennas. The method is computationally highly efficient because the signal processing is done on the time-domain OFDM signals. I.e., the IFFT/FFT blocks are required only once, regardless of the number of diversity branches.**

I. INTRODUCTION

Coded orthogonal frequency division multiplexing (OFDM) systems exploit the frequency-diversity of wide-band radio channels to allow for robust data communications over frequency-selective Rayleigh fading radio channels [1]. The coded data stream is therefore well spread over the signal bandwidth, using an interleaving scheme in the frequency-direction.

The channel's frequency-selectivity, and thus the potential frequency-diversity, depends on the delay spread of the channel. The longer the delay spread, the more fades are present per bandwidth, which is of advantage for the coded and interleaved transmission scheme, because errors occur more independently [2].

The proposed diversity schemes aim to improve the performance in the opposite situation. If the channel delay spread is very short, then the whole OFDM signal is faded equally (i.e., the channel is flat), which induces long error bursts that are hard to correct. There is not sufficient frequency-selectivity in this case to be exploited by the coding and interleaving schemes. Using a set of transmit or receive antennas, the presented diversity schemes can randomize the channel response and thereby increase the frequency diversity. In order to achieve the randomization, cyclic delays are introduced to the effective parts of the OFDM symbols. This principle can be applied at the transmitter and/or at the receiver. At the receiver, the channel transfer functions of individual diversity branches can be estimated and employed for optimizing the diversity combining.

The computational cost of the proposed schemes is very low, as all the signal processing needed is performed on the time-domain OFDM signals; i.e., the discrete Fourier transforms do not need to be duplicated.

The rest of this paper is organized as follows. Section II describes the diversity techniqu for the transmitter and for the receiver. An illustration of its way of operation is given in Section III, together with some design considerations for the delay times. In Section IV, performance results are presented, which have been derived from computer simulations. The impact of the diversity scheme on synchronization and channel estimation algorithms is discussed in Section V, followed by conclusions and recommendations.

II. DESCRIPTION OF THE DIVERSITY SCHEMES

A. Diversity Scheme for the Transmitter

The novel transmitter diversity technique is depicted in Figure 1. Up to the inverse FFT (IFFT), which is used to modulate data constellations on the OFDM sub-carriers and whose output is the time-domain OFDM signal, a conventional OFDM system is present. In order to generate signals for a number of transmit antennas, cyclic delays (of n_i samples or τ_i seconds) are introduced to the (effective) FFT-parts of the OFDM symbols. A cyclic delay means that the n_i samples shifted beyond the effective part are transmitted in the beginning of that part of the symbol (see Figure 2). In

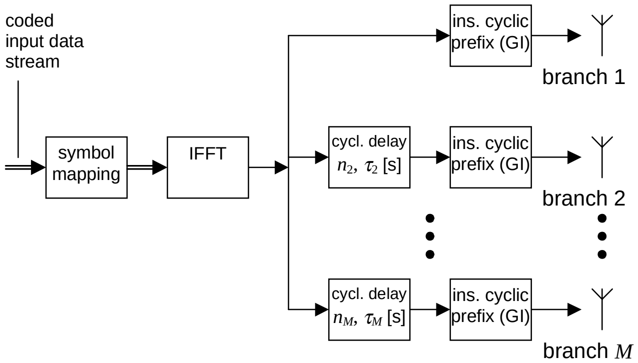

Figure 1: Diversity technique using cyclic delays for the transmitter.

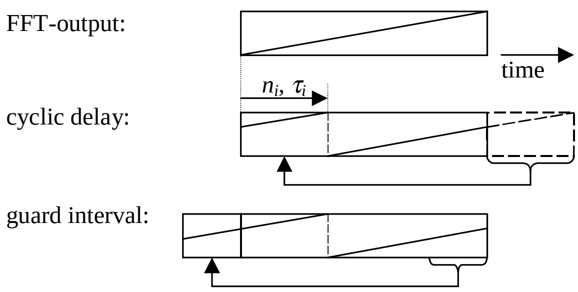

Figure 2: Applying a cyclic delay to the effective part of the OFDM symbol.

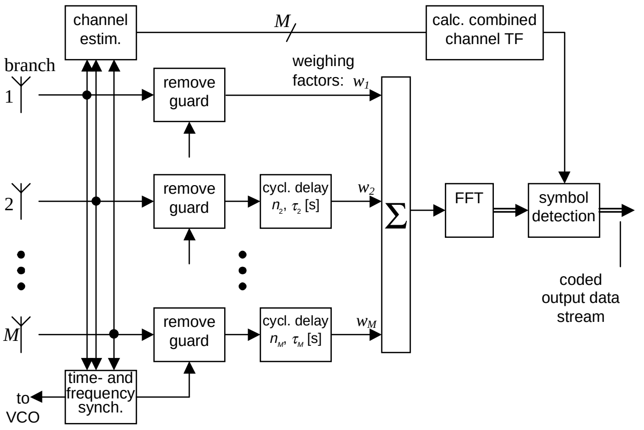

Figure 3: Diversity technique using cyclic delays and weighting factors for the receiver.

the conventional manner, the cyclic prefix (guard interval) is transmitted prior to the effective part.

The method presented is similar to the delay diversity scheme described in [3]. The cyclic delays allow for much longer delays, however, which are otherwise limited to fractions of the guard interval period to avoid inter-symbol-interference. This fact is paramount for applying the proposed technique at the transmitter, where no information is available on the length of the current channel's impulse response.

B. Diversity Scheme for the Receiver

Similarly, cyclic delays can be applied to the OFDM signals received via multiple antennas in order to perform diversity combining at the OFDM receiver, prior to the FFT (see Figure 3). Utilized at the receiver, the delay times can be adapted (optimized) based on individual channel estimates for each diversity branch. Moreover, weighing factors $\{w_i\}$ can be applied to allow for more flexibility. (A pre-FFT diversity scheme based on such weighing factors, but without delays, is analyzed in [4]). The optimization of the parameters is subject for further research. Since the channel transfer functions are (usually) not available at the transmitter, such an optimization is not possible there. Therefore, the application of weighing factors at the transmitter is less promising, although it is generally possible.

The operation of the diversity schemes, and some design considerations for the delay times (for a non-adaptive scheme) are discussed in the following section.

III. Operation of the Method and Selection of the Delay Times

A. Illustration

Following the idealized OFDM system model, the sub-carriers are attenuated and phase distorted according to the channel transfer function (TF) [5]. The diversity scheme randomizes the TF of the composite channel as follows. The TFs, $H_i(f)$, (time-variability is neglected) of a Rayleigh fading channel are correlated, zero-mean, complex Gaussian random processes. The superposition of the channel

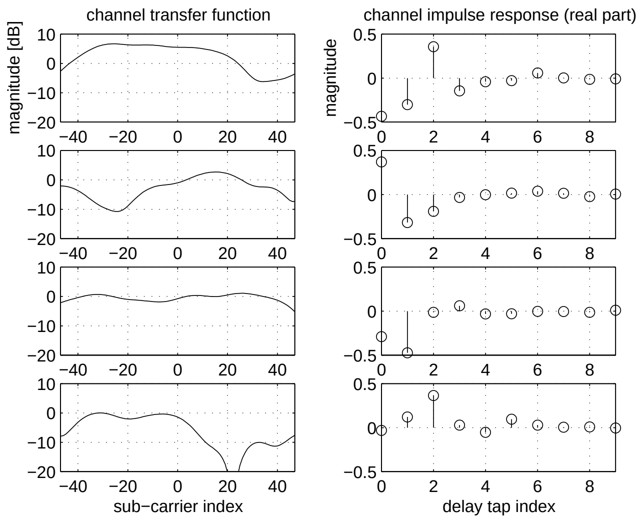

Figure 4: Four (independent) channel realizations. The left-hand side depicts the channels' magnitude transfer functions; the right-hand side illustrates the real-parts of the impulse responses.

TFs of individual antennas is therefore also a Rayleigh channel, as the sum of zero-mean Gaussian processes is yet another zero-mean Gaussian process. The correlation properties are altered, however, by the cyclic delays introduced. The following equation gives the TF for the composite channel. It is seen that the cyclic delays introduce progressive phase rotations to the TFs.

$$H_\Sigma(f) = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} H_i(f) e^{-j2\pi\tau_i f} \ . \qquad (1)$$

The normalization by $1/\sqrt{M}$ is introduced to keep the transmission power constant, when the method is used at the transmitter. At the receiver, this factor can account for the channel noise that adds up incoherently. The noise for the combined channel can then be considered equal to the noise of a single channel. Note that the weighing factors are not incorporated in this brief analysis.

The correlation properties of the composite channel are investigated in the following sub-section. Here, we firstly illustrate the principle of the diversity technique, and we discuss the necessity that signals of *multiple* antennas are combined and that delays are introduced.

Figure 4 depicts the transfer functions and impulse responses of four independent frequency-selective channels. Having short impulse responses, their frequency-selectivity is limited. It is seen that the channels have similar fading characteristics, particularly, their impulse responses have similar length. Superimposing these channels without delays, the sum of channels is just another channel with a similar IR and frequency-selectivity, as seen from the top-row of figures in Figure 5. Nothing is lost or gained in this case.

Introducing the cyclic delays, the composite channel consists of all those IRs, shifted by the respective delay times. This leads to a much-extended overall IR, corresponding to a more random channel TF, as seen from the second row of graphs in Figure 5. Thereby, inter-symbol-interference is avoided due to the cyclic way of applying the delays.

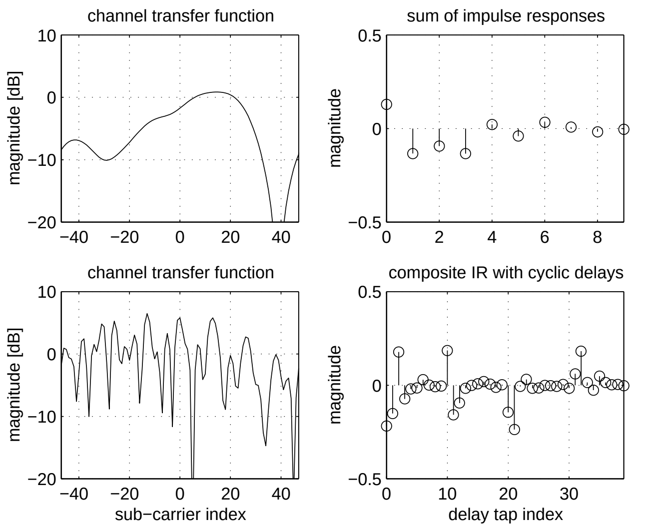

Figure 5: Transfer functions (left-hand side) and impulse responses (right-hand side) of the composite channels. The figures in the first row show the channel of the superposed channel *without* the introduction of cyclic delays. The characteristics of this channel are equivalent to the characteristics of the component channels. In the second row, cyclic delays were applied, leading to a clear randomization of the channel transfer function.

It is not possible to obtain a similar randomization by combining (with delays) multiple copies of the received signal of a single antenna. This would be equivalent to the application of a transversal filter to the received signal. If the filter can be adapted, it is well possible to equalize the transfer function and to get a flat channel response. Unfortunately, the noise level is thereby modified accordingly; therefore nothing is gained. Combining (cyclically) delayed copies of the signal without weighing factors, for instance, means a multiplication of the channel TF by a filter TF that has zeros at certain positions. Clearly, such filtering cannot enhance the performance of an OFDM system.

B. Analysis and Selection of the Delay Times

Let us briefly analyze the correlation properties of the combined channel. It is assumed that the (independent) component channels have similar stochastic properties, expressed by a common spaced-frequency correlation function $\phi_H(\Delta f) = E\{H_i^*(f)H_i(f+\Delta f)\}$. The sum (1) is a sum of zero-mean complex Gaussian random processes, which gives another zero-mean complex Gaussian process, as mentioned above. Accounting for the phase rotations, the spaced-frequency correlation of the composite channel is written as

$$\phi_{H_\Sigma}(\Delta f) = \phi_H(\Delta f)\frac{1}{M}\sum_{i=1}^{M} e^{-j2\pi\tau_i\Delta f} . \qquad (2)$$

The correlation is reduced, because the magnitude of the sum term is less or equal to one [3]. Appropriately selecting τ_i, it is possible to force a zero in this correlation function for the frequency separation corresponding to the separation of subsequent coded bits. Unfortunately, for double the separation, the normalized sum in (2) becomes one again, if two branch diversity is used. The bit at triple that distance

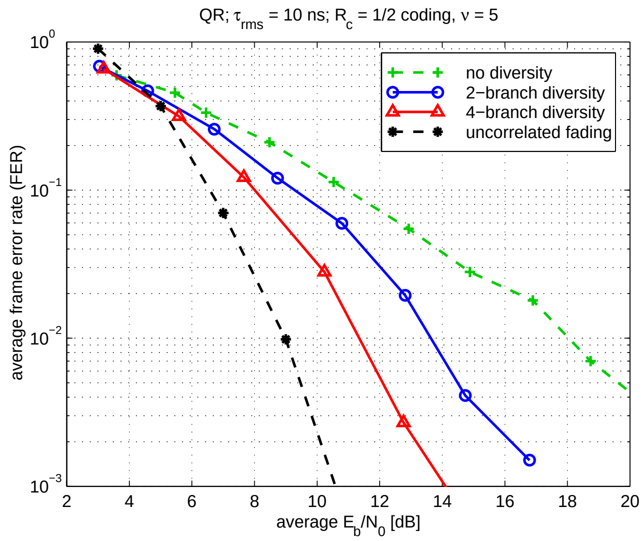

Figure 6: Frame error rate results for a rate-½ convolutional code with a constraint length of five.

is then in a zero again. Additional diversity branches enable the nulling of more subsequent bits.

A very large reduction of the correlation function is also obtained, when zeros are forced to be on directly adjacent sub-carriers, i.e., on sub-carriers separated by $\Delta f = \{F, 2F, \ldots, (M-1)F\}$, where F is the sub-carrier spacing. This makes the correlation function at frequency-separations of integer multiples of F

$$\phi_{H_\Sigma}(kF) = \begin{cases} \phi_H(kF) & \text{if } k = lM \\ 0 & \text{otherwise} \end{cases} , \qquad (3)$$

where $\{k,l\}$ are integer variables. Such a result is obtained for delays

$$\tau_i = \frac{i-1}{MF} \text{ or } n_i = (i-1)\frac{N_{FFT}}{M} . \qquad (4)$$

Note that, when using this parameter set, it is important that the interleaving depth is selected differently to M, otherwise the correlation of the fading on subsequent coded bits is not reduced at all.

IV. Performance

Simulation results of frame-error-rates (FER) (packet length = 57 byte) are given in Figure 6. The OFDM system model assumes perfect synchronization and channel estimation. An OFDM scheme with just 19 data sub-carriers has been simulated (QPSK modulation; FFT-size N_{FFT} = 32), which implies quite low frequency-diversity. The channel's RMS delay spread was about 0.3 samples; the channel was assumed quasi-static during the transmission of the 24 subsequent OFDM symbols carrying on data packet. Delays of {0,4} and {0,2,4,6} samples were introduced for the two and four-branch diversity schemes, respectively. (Those delays yielded zeros in the correlation function at adjacent coded bits, due to the application of a depth four interleaver). For comparison, the FER is also shown without diversity and for the special case that all sub-carriers are faded independently.

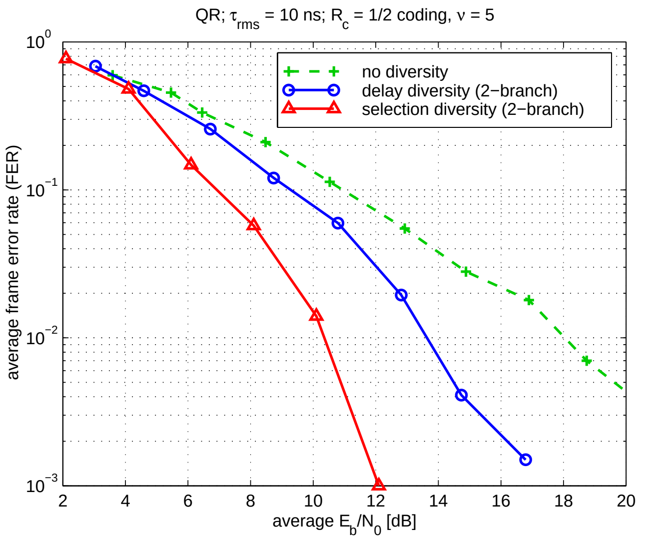

Figure 7: FER-performance of a selection diversity scheme compared with the proposed delay diversity technique. (Two branches.)

It is seen that, at a FER of 0.01, a gain of almost 10 dB is possible for the special case (independent fading), compared with the single antenna result. About 5 and 7 dB are obtained with two- and four-branch diversity, respectively.
Less gain is anticipated over channels that have longer delay spreads and thus more frequency-diversity. The same applies for OFDM systems with a larger bandwidth, as they can exploit the frequency-diversity in the single antenna set-up more efficiently.
Figure 7 compares the performance of two-branch delay diversity with the performance of selection diversity applied at the receiver. Selection diversity adaptively chooses on each sub-carrier the signal constellation of the strongest branch. Due to this adaptivity, about 3 dB gain are obtained, compared to the transmitter diversity scheme. The slopes of the FER-curves are about equal, however.

V. Discussion of the Diversity Schemes

The computational cost of the proposed schemes is very low, as all the signal processing needed is performed on the time-domain OFDM signals; i.e., the discrete Fourier transforms do not need to be duplicated. For comparison, Figure 8 shows an OFDM receiver that can utilize *conventional* diversity techniques, as for instance, selection diversity, maximal ratio combining, or equal gain combining. Note that all blocks of an OFDM receiver including the fast Fourier transform (FFT) must be present M-times, where M is the number of diversity branches. For each sub-carrier, the output values of the M FFT blocks are combined, which means a significant increase in complexity compared to the single-antenna receiver. The complexity-increase for the novel techniques described in this paper is therefore much smaller.
A considerable performance improvement is the main advantage of the techniques investigated, as seen from the performance results. Basically, the transmitter diversity schemes can be employed without modifying the receiver. However, care must be taken with synchronization and channel estimation techniques and to some extent with the

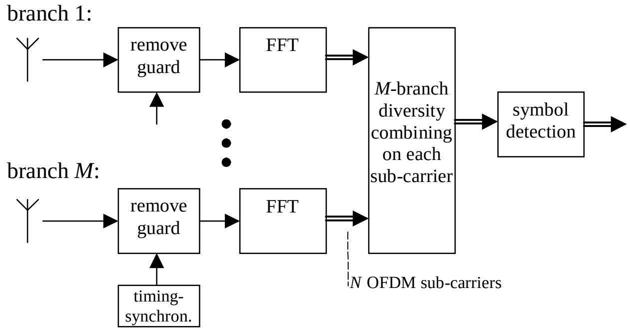

Figure 8: Block diagram of an OFDM receiver allowing conventional diversity techniques on each sub-carrier.

interleavers. That is, the parameters of all system components should be carefully chosen to minimize interference effects among them. The following points have to be considered.
In Schmidl's method [6], (frame) timing-synchronization is done by looking for a unique training symbol, which carries information only on the even-indexed sub-carriers. The odd sub-carriers are zero. Such symbols do not occur in a conventional OFDM signal, unless they are explicitly inserted. Using the two-branch delay diversity technique with delays $\{\tau_i\}$ selected according to (4), signals with very similar properties may be evident at the receiver, however, because all the odd and all the even sub-carriers remain correlated. (Only the odd sub-carriers are de-correlated from the even ones.) Therefore, all odd sub-carriers may be attenuated simultaneously – on a channel with short delay spread –, while the even sub-carriers are strong. Such a, not unlikely, composite input signal may be miss-interpreted as the training symbol. If known data is modulated on the training symbol, the frame start can be confirmed by demodulation.
Timing synchronization offsets introduce a progressive phase rotation (see [5]). This progressive phase rotation can be accurately determined from a known training symbol, yielding an estimate of the residual timing-offset after frame synchronization [7], [8]. Again, high correlation among the sub-carriers has to be assumed in order to achieve good performance in presence of a frequency-selective channel. This correlation is largely destroyed, however, by the diversity technique. A possible remedy for this issue is to calculate the timing-offset estimate over the still-correlated sub-carriers. (The ones spaced by M sub-carriers, if the $\{\tau_i\}$ are selected according to the criterion given in eq. (4)).
Channel estimation schemes sometimes exploit the correlation between the channel coefficients of adjacent sub-carriers in order to reduce the noise floor of the channel estimates or in order to estimate the channel on sub-carriers that do not carry pilot symbols. Eliminating this correlation by the suggested diversity techniques, it is clear that such principles would perform sub-optimally or fail. For instance, the first of the two training symbols proposed by Schmidl [6] for synchronization could be used for channel estimation. As this training symbol does not carry data on the odd sub-carriers, the estimates on the adjacent sub-carriers can be averaged to obtain the channel estimates. It is obvious that this interpolation fails if correlation among

adjacent sub-carriers lacks.

Applying the diversity technique at the receiver, the above issues can be avoided, since channel estimation and synchronization can be performed on the signals of individual antennas. An additional step is then required to calculate the channel estimate for the combined channel.

The amplitude distribution of the combined channel is another point to be considered. If the component channels are Rayleigh distributed then the sum of the channels will be Rayleigh distributed as well, and performance gain is obtained due to the enhanced frequency-diversity. The depth of the fades won't be influenced.

On channels with shallower fades, however, – for instance Ricean channels with considerably high K-factors –, the performance is generally improved due to the lower probability of "deep fades". Applying the proposed diversity schemes in this case, the combined channel (see eq. (1)) would have deeper fades due to the randomization. A possible performance gain by the non-adaptive techniques is therefore questionable.

VI. Conclusions and Recommendations

Considerable diversity gain at low additional complexity can be obtained with the proposed delay diversity schemes for OFDM, which employ cyclic delays to avoid inter-symbol-interference. Applied at the transmitter (where no channel information is available), the frequency-diversity of the channel is increased. Applied at the receiver, channel estimates for individual diversity branches can be employed to optimize the diversity combining. This is subject for further investigations.

The possible impact of the transmitter diversity scheme on synchronization and channel estimation has been discussed. It has been concluded that the parameters of all system components must be carefully designed in order to minimize interference effects.

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