Signal Processing and Speech Communication Lab.
Graz University of Technology

FDC

Third Homework Assignment for Fund. of Digital Commun.

Name MatrNr.

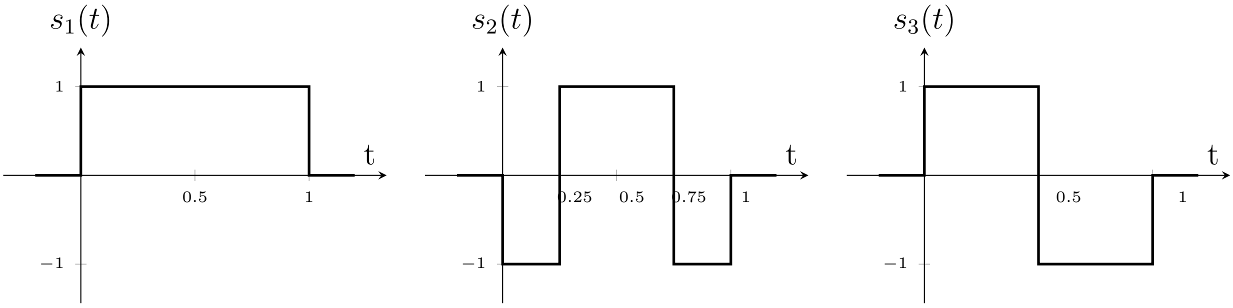
Please hand in your homework no later than **Feb. 3, 2017**, 16:00, at our mailbox, Inffeldgasse 16c, ground floor. Please use this assignment sheet as a cover page, filling in your name.
Please send your simulation code (*.m) and simulation protocol (*.pdf) via email (*hw1.spsc@tugraz.at*).
Clearly indicate your name and MatrNr in the subject of the mail.

Problem 3.1 (13 Points)

A binary transmission system is defined by two Gaussian likelihood functions $f(r|s_i)$, $i = 1, 2$, with mean values $s_1 = -1$ and $s_2 = 1$, respectively, and variances $\sigma_n^2 = 0.5$. The symbols s_i are sent with a-priori probabilities of $P(s_1) = 0.8$ and $P(s_2) = 0.2$.
(a) (3 points) Give the equations for the likelihood functions and illustrate them.
(b) (2 points) Determine metrics for an optimum decision according to the maximum a-posteriori (MAP) criterion and sketch them as a function of r.
(c) (4 points) Determine a threshold value for the MAP decision and indicate it in the figure.
(d) (4 points) Determine the error probability for the MAP decision.

Problem 3.2 (20 + 3 Bonus Points)

Three symbols m_1, m_2, and m_3 are to be transmitted over an AWGN channel with double-sided noise power-spectral density $N_0/2$. The *prior* probabilities of the symbols are $P(m_1) = 0.05$, $P(m_2) = 0.35$ and $P(m_3) = 0.6$. The symbols are transmitted using their corresponding signals, which are given as



(a) (3 points) Show that the dimensionality of the signal space is $N = 3$ and that the signals $s_1(t)$, $s_2(t)$, and $s_3(t)$ are an orthonormal basis. Compute the corresponding signal vectors $\mathbf{s}_m$, $m = 1, 2, 3$ and sketch them.
(b) (2 points) Determine the structure of the optimum receiver (correlation type).

(c) (4 points) Formulate the likelihood function $f(\mathbf{r}|\mathbf{s}_m)$ for AWGN. Formulate the MAP detection criterion for this signaling scheme for detecting the m-th symbol! (Use the log likelihood function and simplify the criterion as much as possible.) Compute also the ML criterion!

(d) (2 points) For ML detection of higher-order modulation schemes the symbol error probability can be approximated using

$$P_s \approx \bar{N}_e \mathrm{Q}\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right),$$

where $\bar{N}_e$ denotes the average number of nearest neighbors (in signal space), and $d_{\min}$ denotes the minimum distance to those neighboring signal vectors. How many nearest neighbors (in average) does the given modulation scheme have? Compute the minimum distance $d_{\min}$ as a function of the signal energy $\mathcal{E}_{av}$. How does the approximation look like for the given modulation scheme with respect to the signal energy ($\mathcal{E}_{av}$) to noise power-spectral density (N_0) ratio $\frac{\mathcal{E}_{av}}{N_0}$.

(e) (9+3 points) Write an Octave/Matlab simulation which computes the symbol error probability with respect to $\frac{\mathcal{E}_{av}}{N_0}$ in the interval -4 dB $\leq \frac{\mathcal{E}_{av}}{N_0} \leq 12$ dB using the MAP and ML criteria. The signal space model of the receiver output is given as

$$\mathbf{r} = \mathbf{s}_m + \mathbf{n}.$$

The simulation should generate at least 10^6 realizations according to this signal model and should determine the error probability by comparing the simulated detector output and the known data transmitted. Show your numerically computed symbol error probability curve using the MAP and ML decision criteria in a plot and compare it with the corresponding theoretical approximation derived in (d). Attach a hardcopy of your results to your homework and send your scripts to hw1.spsc@tugraz.at. Which criterion leads to the better symbol error rate and why?

(Hint: Note that the simulation may take a while (depending on the efficiency of the implementation: from less than 10 seconds up to more than ten minutes.(**For efficient programming you can earn + 3 Bonus Points**)).