Signal Processing and Speech Communication Lab.
Graz University of Technology

FDC

First Homework Assignment for Fund. of Digital Commun.

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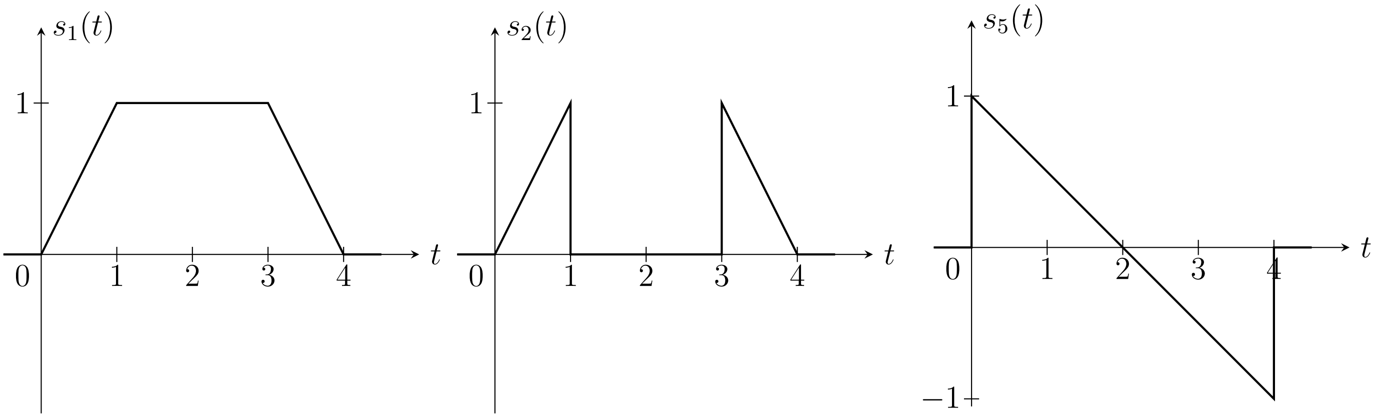
Please hand in your homework no later than **Nov. 27, 2017**, 15:00, at our mailbox, Inffeldgasse 16c, ground floor. Please use this assignment sheet as a cover page, filling in your name.

Problem 1.1 (2 Points)

Are the four (body-) diagonals of a cube orthogonal? Justify your answer (you do not need to compute anything, but rather argue)!

Problem 1.2 (13 Points)

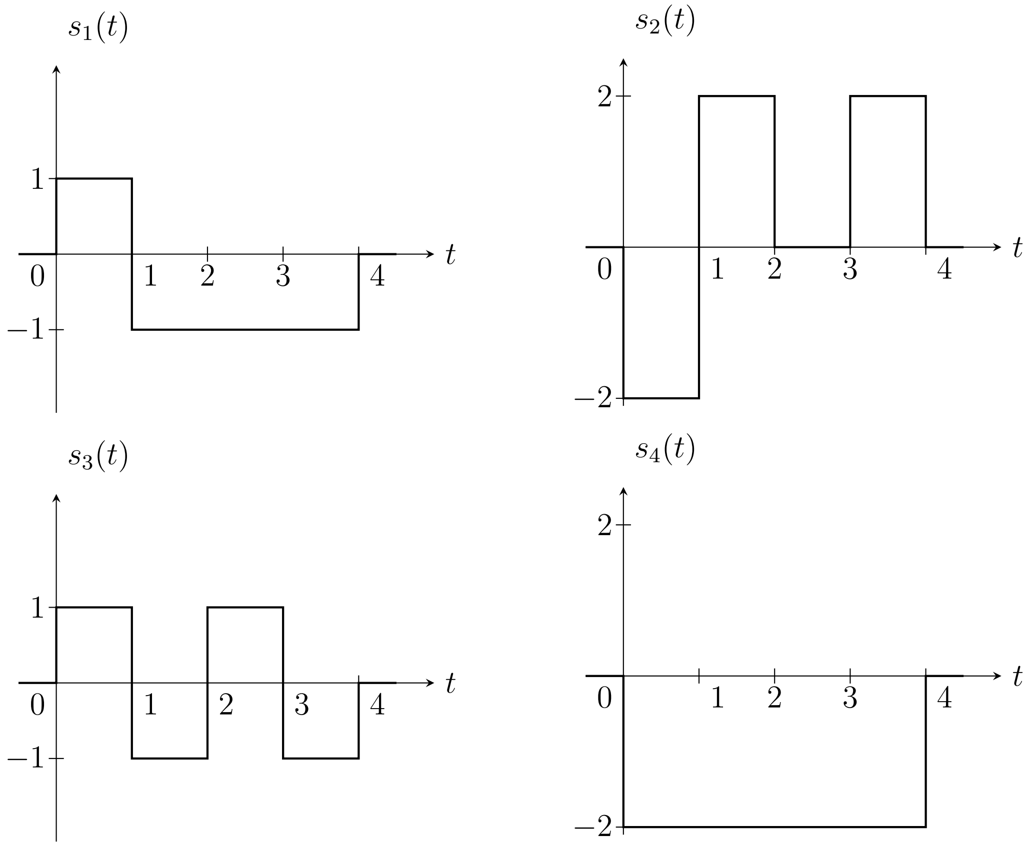
Given three signals $s_1(t)$, $s_2(t)$ and $s_5(t)$:



(a) Find the norms of $s_1(t)$ and $s_2(t)$.
(b) Find the inner product of these two signals in a linear space. What is the angle between the two signals? Sketch the vectorial representation of the signals!
(c) Find the norm of the signal $s_3(t) = s_1(t) + s_2(t)$. Sketch the signal $s_3(t)$.
(d) Find a signal $s_4(t)$ that is *in* the subspace spanned by $s_1(t)$ and $s_2(t)$ ($\mathcal{S} = \text{span}\{s_1(t), s_2(t)\}$) and is orthogonal to $s_3(t)$.
(e) Find and sketch the signal that lies in the subspace spanned by $s_1(t)$ and $s_2(t)$ and that is closest to $s_5(t)$.
(f) Compute and sketch the signal which represents the projection error $e_5(t)$, resulting from projecting $s_5(t)$ onto the subspace spanned by $s_1(t)$ and $s_2(t)$.

(g) Find an orthonormal basis $\{\psi_1(t), \psi_2(t)\}$ for $\mathcal{S}$. Hint: consider the geometric representation of the signals $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t) \in \mathcal{S}$ to complete this task without cumbersome computations.
(h) Compute the coefficient vectors to represent the signals $s_1(t)$ to $s_4(t)$ in this basis (i.e. find the orthonormal basis expansions of the signals).

Problem 1.3 (8 Points)



(a) Using the Gram-Schmidt orthogonalization, find an orthonormal basis for the four waveforms $s_1(t), s_2(t), s_3(t)$, and, $s_4(t)$.
(b) What is the dimensionality of the space $\mathcal{S}$ span by $s_1(t), s_2(t), s_3(t)$, and, $s_4(t)$?
(c) Determine the signal vectors $\mathbf{s}_i$ for this orthonormal basis.

Problem 1.4 (10 Points)

The signal $x(t)$ is applied to a linear, time-invariant system with impulse response $h(t)$, where

$$x(t) = \begin{cases} 1 & \text{for } |t| \leq T/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(t) = \begin{cases} 1 & \text{for } |t| \leq T/2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Sketch the signals $x(t)$ and $h(t)$. The convolution integral can be used to determine the output signal $y(t)$ of the LTI system. Define this convolution integral in its generic form.

(b) Determine and sketch the output signal $y(t)$.
(c) Repeat task (b) with the signal $x(t) = y(t)$ as input to the LTI system $h(t)$.
(d) Using numerical software (e.g. Octave, Matlab, etc.) evaluate the convolution of a cascade of 10 LTI systems with impulse response $h(t)$ when excited with the input signal $x(t)$ (Hint: think about the support of the convolution and use $T = 2$ s). You do not need to hand in your code, but please add a plot of your output signal $y_{10}(t)$ at the output of the cascade. What can you observe?