Analysis of Position-Related Information in Measured UWB Indoor Channels

(Convened Session)

Paul Meissner and Klaus Witrisal
Graz University of Technology, Graz, Austria, Email: {paul.meissner, witrisal}@tugraz.at

Abstract—Conventional radio-based indoor localization systems often only make use of the direct signal path between an agent and the anchor nodes. Therefore, performance can deteriorate in non-line-of-sight situations, even though reflected multipath-components carry useful location-dependent information. Using ultra-wideband signals, these components become resolvable. Bringing together previously obtained results on the Cramér-Rao bound on the position error and measurement data along a reference trajectory, we present a methodology for quantifying the position-related information contained in UWB indoor channels. An explicit model for the diffuse part of the channel enables to evaluate its importance realistically. Experimental results verify prior theoretical work. Important parameters and their influence on the system performance are discussed.

I. INTRODUCTION

For multilateration-based indoor localization, ultra-wideband (UWB) signals are the method of choice, especially due to the fact that the line-of-sight component can be separated from the reflected multipath components (MPCs). In non-line-of-sight (NLOS) situations however, these systems can still show large errors due to biased range estimates. Proposed solutions try to e.g. identify and discard NLOS measurements, which implies loosing all geometric information that is embedded in the reflected components. In prior work (see [1] and references therein), we proposed an indoor localization method that explicitly makes use of multipath propagation by mapping MPCs to virtual anchors (VAs) that are positioned according to a known floor plan.

Recently, we have derived the Cramér-Rao lower bound (CRLB) for this multipath-assisted indoor navigation and tracking (MINT) scenario [2]. In contrast to other work on this topic, like [3] and [4], we explicitly model diffuse multipath (DM) using a stochastic process. With this, we are able to evaluate the influence of DM on the localization performance, which is important as the density of diffuse components can be high in indoor scenarios. The often-used model for the UWB channel as a sum of reflected copies of the transmit signal plus noise seems to be inappropriate for this purpose. This paper aims at a verification of these theoretical results using data from an extensive indoor measurement campaign [5].

We present a framework for the quantification of position-related information in UWB channel measurements. The results are used to verify the theoretical work in [2]. Also, they provide important performance considerations for localization schemes making use of multipath propagation, like [6] or [7], for example.

II. SCENARIO AND SIGNAL MODEL

Fig. 1 shows the scenario that is analyzed in this paper and where the measurement campaign was performed [5]. The position of the single anchor node \( p_1 \) is mirrored with respect to the reflecting surfaces using the known floor plan. This results in VA nodes at positions \( p_k = [x_k, y_k]^T, k = 2, \ldots, K \) [8]. These can be exploited for multipath-assisted indoor navigation and tracking (MINT) [2].

The UWB channel was measured at 381 points, spaced by 10 cm with a Rhod & Schwarz ZVA-24 vector network analyzer over a frequency range of 3.1-10.6 GHz. Using pulse
shaping with a “transmit signal” \( s(t) \), the signal at position \( p_i \) is modeled as [2]

\[
 r_i(t) = \sum_{k=1}^{K_i} \alpha_{k,i} s(t - \tau_{k,i}) + n_i(t) \tag{1}
\]

which consists of \( K_i \) deterministic MPCs with amplitudes \( \{\alpha_{k,i}\} \in \mathbb{C} \) and propagation delays \( \tau_{k,i} = \frac{1}{c}||\mathbf{p}_i - \mathbf{p}_k|| \), where \( c \) is the propagation velocity. We assume that these MPCs are caused by specular reflections at flat surfaces, i.e., they are modeled by the VAs. The noise term \( n_i(t) \) consists of both measurement noise, \( w_i(t) \), and the convolution of the transmitted signal with the DM, which is modeled by a random process \( \nu_i(t) \), i.e.

\[
 n_i(t) = \int_{-\infty}^{\infty} s(\lambda) \nu_i(t - \lambda) d\lambda + w_i(t). \tag{2}
\]

Here, \( w_i(t) \) denotes additive white Gaussian noise with two-sided power spectral density \( N_0/2 \). The lower part of Fig. 1 shows the measured channel impulse responses (CIRs) along the trajectory. We can observe the structure of the deterministic components, that bears information for localization [9].

The aim of this paper is a verification of the results on the performance bounds for MINT [2] using the measured CIRs along the trajectory. One major challenge in this regard is the lack of sufficiently many (and densely spaced) measurement points to reliably estimate channel statistics. E.g., in [2], uncorrelated scattering is assumed, which means that the process \( \nu_i(t) \) is modeled by an ACF \( K_{\nu_i}(\tau, u) = \mathbb{E}[\nu_i(\tau)\nu_i(\tau + u)] = S_\nu(\tau)\delta(\tau - u) \). According to this model, the power delay profile \( S_\nu(\tau) \) does not consider spatial variations as it does not depend on the position \( l \). However, it is clear that in the scenario in Fig. 1, visibility regions of the VAs and hence propagation characteristics change severely, even over a distance of just a few meters [5]. Hence, we restrict our analysis to only \( L = 21 \) measurement points \( p_i \) for \( l = L_1, \ldots, L_2 \) (see Fig. 1, for comparison with [5], \( L_1 = 60 \) and \( L_2 = 80 \)), and stack the corresponding measurements in the vector

\[
 \mathbf{r}_{L_1,L_2}(t) = [r_{L_1}(t), \ldots, r_{L_2}(t)]^T. \tag{3}
\]

### III. Bounds on the Position Error

In [2], the Cramér-Rao lower bound on the position error for multipath-assisted localization has been derived by blockwise inversion of the Fisher information matrix (FIM) \( \mathbf{J}_\mathbf{\theta} \) [3] for the parameter vector \( \mathbf{\theta}_l = [\mathbf{p}_l^T(\alpha^H(\nu)) (\alpha(\nu))^T]_l \) that contains the position and the real and imaginary components of the MPC amplitudes \( \alpha_{k,i} \). The equivalent FIM (EFIM) [3] for the position \( \mathbf{p} \) has been derived as

\[
 \mathbf{J}_{\mathbf{p}} = \frac{8\pi^2\beta^2}{c^2} \sum_{k=1}^{K_1} \text{SINR}_{k,l} \mathbf{J}_f(\phi_{k,l}) \tag{4}
\]

where no path overlap, i.e., orthogonality of the signals from different VAs, is assumed. Ref. [2] also derives \( \mathbf{J}_{\mathbf{p}} \) for path overlap situations. As we will discuss below, this situation leads to considerable practical difficulties, and is not considered here. In (4), \( \beta^2 \) is the effective bandwidth of \( s(t) \) and \( \mathbf{J}_f(\phi) \) is the ranging direction matrix [3], with one eigenvector in the direction \( \phi_{k,l} \), which is the angle from \( \mathbf{p}_k \) to \( \mathbf{p}_l \). The signal-to-interference-plus-noise ratio (SINR) of the \( k \)-th MPC at position \( \mathbf{p}_l \) is

\[
 \text{SINR}_{k,l} := \frac{|\alpha_{k,l}|^2}{N_0 + T_s S_\nu(\tau_{k,l})}. \tag{5}
\]

In (5), the influence of the individual terms of the CIR (c.f. (1)) is shown: The weights \( w_k \) account for the influence of DM via its power delay profile \( S_\nu(\tau) \). We observe that the DM is scaled by \( T_s \), the bandwidth inverse. Again we note that with limited measurement points, \( S_\nu(\tau) \) or a reliable estimate of it will not be available, and due to the \( 10 \) cm spacing of the \( \mathbf{p}_l \), the spatial variation can not be neglected. Hence, instead of estimating \( \text{SINR}_{k,l} \) for each \( p_i \), we will estimate an average \( \text{SINR}_k \) for each VA, restricted to our measurement set in \( \mathbf{r}_{L_1,L_2}(t) \).
IV. ESTIMATION OF POSITION-RELATED PARAMETERS

For the contribution of one deterministic MPC at the position \(p_k\), the log-likelihood function that results in the EFIM in (4) can be expressed as [2]

\[
\ln f(r(t)|\theta_{k,l}) \propto \frac{2}{N_0} \int_0^T \Re \left[ r(t) \alpha_{k,l} w_k^* \dot{s}(t - \tau_{k,l}) \right] dt - \frac{1}{N_0} \int_0^T \left| \alpha_{k,l} w_k s(t - \tau_{k,l}) \right|^2 dt \tag{6}
\]

where \(\theta_{k,l} = [\alpha_{k,l}^T \alpha_{k,l}^T]^T\), \(T\) is the observation interval and the delay \(\tau_{k,l}\) depends on the respective VA position \(p_k\) as

\[
\tau_{k,l}(p_k) = \frac{1}{c} ||p_k - p_l||. \tag{7}
\]

Equation (6) is the log of the likelihood ratio of a signal \(s(t)\), parameterized by \(\theta_{k,l}\), in Gaussian noise [10]. The scaling factors \(w_k\) result from a whitening operation used to account for the colored noise in \(v(t)|2\).

To evaluate the suitability of a Gaussian model for the noise term \(v(t)|2\) given in (2), we use the channel analysis method in [5]. We subtract the influence of the deterministic MPCs caused by the VAs from \(v(t)|2\). The remaining signal is an estimate for the combined noise term \(\tilde{v}(t)|2\) [11]. Fig. 2 shows the procedure for several trajectory positions and a bandwidth of 2 GHz. We observe that the DM is often especially pronounced after deterministic MPCs, i.e. those show a “diffuse tail” associated to them [12]. These components seem to carry significant energy which severely influences multipath-assisted localization schemes [1].

Although elaborate statistical modeling is out of the scope of this paper, we exemplarily analyze \(\tilde{v}(t)|2\) for a certain part of the channel. For this purpose, we extract \(\tilde{v}(t)|2\) for a range of 2 ns after the MPC from VA 10 (marked in Fig. 2). A histogram of its real part, together with an ML-fit of a Gaussian distribution, is shown in Fig. 3. Although both a slight concentration of values in the vicinity of zero and outliers with quite large values can be observed, a zero-mean Gaussian model might be a reasonable assumption.

A. Maximum-Likelihood Localization of VAs

We assume that in a realistic scenario, the floorplan is not known exactly and therefore, the VA positions \(p_k\) are subject to uncertainty. Hence, as a first step in the data extraction process, we re-estimate the locations of the VAs according to the model in (6). In a small region around the geometrically expected VA position \(p_k\), we search for the VA position that provides a best fit to the received signal. For each candidate point \(p_k\) within this region, we construct a template signal

\[
\tilde{s}_{k,l}(t) = \tilde{a}_k s(t - \tilde{\tau}_{k,l}(p_k)) \quad \text{with} \quad ||s(t)|| = 1 \tag{8}
\]

where \(||s(t)||\) denotes the norm of \(s(t)\). The complex amplitude of the template, \(\tilde{a}_k\), is kept constant with respect to \(l\), as it is a nuisance parameter in this estimation. Assuming noncoherent processing, we choose \(\tilde{a}_k\) as the mean of the absolute value of \(r(t)|2\) over the trajectory, i.e.

\[
\tilde{a}_k = \frac{1}{T} \sum_{l=1}^{L} |r_l(\tilde{\tau}_{k,l})| \tag{9}
\]

where \(\tilde{\tau}_{k,l}\) is the expected delay according to the floorplan. Assuming independent measurements at different trajectory positions, the log-likelihood function is given as

\[
\ln f(r_{l_1:l_2}(t)|p_k) \propto \frac{2}{N_0} \sum_{l=1}^{L_2} \int_0^T |r_l(t)\tilde{s}_{k,l}(t)| dt - \frac{1}{N_0} \int_0^T |\tilde{s}_{k,l}(t)|^2 dt. \tag{10}
\]

It is important to restrict the computation of (10) to those measurement positions \(p_l\) where the \(k\)-th VA is expected to be visible, and where additionally no path overlap with any other VA occurs. The latter is important because this processing allows no “distribution” of MPC energy to different sources at the same delay. However, in the selected part of the trajectory, path overlap only occurs for some positions between VAs 15 and 24, if a bandwidth of 2 GHz or below is used. The ML estimate of the position of the \(k\)-th VA is then computed as

\[
\hat{p}_k = \text{arg max}_{\hat{p}_k} \ln f(r_{1:l_2}(t)|\hat{p}_k) \quad \text{where} \quad ||\hat{p}_k - p_k|| \leq \hat{d} \tag{11}
\]

where \(\hat{d}\) is the radius of the uncertainty region around the expected \(p_k\) and is chosen as 20 cm.

B. Estimation of Path Amplitudes and SINR for VAs

In this step, the complex amplitudes of the MPCs are estimated. To this end, we project the received signal onto the energy-normalized transmit signal, shifted to the ML estimate \(\tilde{\tau}_{k,l}\) of its respective path delay, i.e. \(\tilde{s}_{k,l}(t) = s(t - \tilde{\tau}_{k,l})\). The projection is then

\[
\tilde{a}_{k,l} = \int_0^T r(t)\tilde{s}_{k,l}^*(t) dt. \tag{12}
\]

With the assumptions that \(s(t) \approx 0\) for \(|t| > T_s\), no path overlap, and that the ML estimate of the delay is exact, i.e. \(\tilde{\tau}_{k,l} = \tau_{k,l}\), this reduces to

\[
\tilde{a}_{k,l} = \alpha_{k,l} + \int_0^T [s(t) * \nu(t) + w(t)]\tilde{s}_{k,l}^*(t) dt = \alpha_{k,l} + \nu_{k,l} + w_{k,l} \tag{13}
\]

where \(\ast\) denotes convolution. Hence, the estimated amplitude \(\tilde{a}_{k,l}\) is a sum of the true amplitude \(\alpha_{k,l}\) and the projections of the DM and the noise on the estimated template for the \(k\)-th MPC. As there is no way of finding these components individually based on observing \(\tilde{a}_{k,l}\) without additional knowledge, we proceed by directly estimating the corresponding average SINR\(\tilde{\gamma}\) for the \(k\)-th VA.

For this, we assume that the deterministic amplitude \(\alpha_{k,l}\) is a constant with respect to \(L\) measurement points. This implies that all variations in \(\tilde{a}_{k,l}\) are caused by DM and noise.
To allow this assumption, we correct the deterministic distance dependence of the $\alpha_{k,l}$ by multiplying them with a factor $f_{corr,k} = \sqrt{F_k(l_0)/F_k(l)}$, where $F_k(l)$ is the Friis equation for the free space loss between the VA at $p_k$ and the position $p_l$. The position $p_{l0}$ is the one closest to the average distance between $p_k$ and all $p_l$.

As $\alpha_{k,l}$ consists of the deterministic complex amplitude $\alpha_k$, which is superimposed by Gaussian noise, it is reasonable to model its absolute value as $|\alpha_{k,l}| \sim \text{Rice}(s = |\alpha_k|, \sigma^2)$, where $\sigma^2$ denotes the variance of the noise. We observe that the Ricean K-factor, $s/2\sigma^2$, is a reasonable estimate for SINR$_k$.

The estimated complex amplitudes $\hat{a}_{k,l}$ can be written as

$$\hat{a}_{k,l} = \alpha_k e^{j\phi_{k,l}} + N_l \quad \text{where} \quad N_l \sim \mathcal{CN}(0,2\sigma^2) \quad (14)$$

where the random variable $N_l$ denotes contributions from DM and noise. Multiplying (14) with $e^{-j\phi_{k,l}}$ yields

$$\hat{a}_{k,l} e^{-j\phi_{k,l}} = \alpha_k + N_l e^{-j\phi_{k,l}} = X_1 + jX_2 \quad (15)$$

where

$$X_1 \sim \mathcal{N}(0,|\alpha_k|^2) \quad \text{and} \quad X_2 \sim \mathcal{N}(0,2\sigma^2) \quad (16)$$

Hence, the random variable $Y = |\hat{a}_{k,l}|^2$ will follow a non-central $\chi^2$-distribution with $n = 2$ degrees of freedom [13], whose parameter $s^2$ is the sum of the means of the underlying Gaussians, i.e. $s^2 = E[X_1]^2 + E[X_2]^2 = |\alpha_k|^2$. Mean and variance of $Y$ are given as

$$E[Y] = n\sigma^2 + s^2 \quad (17)$$

$$\text{var}[Y] = 2n\sigma^4 + 4\sigma^2 s^2 \quad (18)$$

which means that $s^2$ and $\sigma^2$ and hence the desired estimate for the SINR$_k$ can be computed from moment estimates of (17) and (18), which we denote with $m_{1,Y}$ and $m_{2,Y}$, respectively. We obtain

$$\text{SINR}_k \approx \frac{1}{m_{1,Y}/m_{2,Y} - 1} \quad (19)$$

where we observe that the estimate of SINR$_k$ can be complex if the variance of the observed $|\hat{a}_{k,l}|^2$ is high. As in these cases the true SINR would be very low, we set it to zero.

V. RESULTS

We performed the steps described in the previous sections over the $L = 21$ trajectory points shown in Fig. 1. The analysis is performed at three different bandwidths, the full FCC bandwidth of $W = 7.5$ GHz between 3.1 and 10.6 GHz, $W = 2$ GHz and $W = 1$ GHz. The latter two both start at 6 GHz. It should be noted that the estimates for e.g. (19) are based on at most $L = 21$ samples, which is due to the previously discussed limitations of the measurement campaign. However, the presented methodology can be used as a framework for future campaigns, as the results below show its validity in principle.

Table I shows the estimated SINR$_k$ for the individual VAs. SINR$_\text{ref}$ indicates the total SINR of all reflected (i.e. all but the LOS) MPCs. As expected from the geometry, we observe a high SINR$_1$, which corresponds to the LOS anchor. For $W = 7.5$ GHz, the SINR$_k$ is high for most VAs, which is due to the fact that their contributions are well resolvable in the received signals. This mostly carries over to $W = 2$ GHz (observe the bandwidth dependence in (5)), but only to a lesser extent to $W = 1$ GHz. Although we exclude path overlap situations in this work, some interference from closely spaced pulses $s(t)$ is visible for e.g. VAs 1 and 10. SINR$_{10}$ is significantly lower for this bandwidth, as the corresponding pulse is already distorted by the LOS component which causes the re-localization of the respective VA to be erroneous. Hence, path overlap situations are identified as an especially important issue for future work.

Fig. 4 shows the position error bound (PEB), i.e. the square root of the trace of $J_{p1}^{-1}$ over the measurement points. The beneficial influence of bandwidth is clearly visible. As in [2], the CRLB is decomposed into orthogonal (principal) components (PC). We see that most of the expected position error can be attributed to one direction (PC0), while in direction of PC1, low errors are expected (c.f. Fig. 5). Over the analyzed part of the trajectory, the PEB is seen to be roughly constant. For $W = 2$ GHz, we observe an increase at $p_{72}$, where VA 15 becomes invisible, increasing the PEB in the corresponding direction.

Fig. 5 finally shows the ten-fold position error standard deviation ellipses for some points on the trajectory. The results are extrapolated for positions $p_l$ with $l > 80$, i.e. the previously estimated SINR$_k$ is used. The geometry dependence is clearly shown, as the bound in direction to the physical anchor is very low. At position $p_{85}$, VA 11 is no longer visible, and the remaining VAs are all located on a vertical line through the anchor. The resulting geometric dilution of precision is well predicted by the CRLB. An additional comparison is done with an Extended Kalman Filter (EKF) algorithm presented in [1]. This EKF can rely on perfect data association (DA) of MPCs to VAs. Analysis of its estimation error covariance matrix reveals a good match with the CRLB, which further verifies its usefulness for performance analysis and prediction of multipath-assisted indoor localization.
VI. CONCLUSION AND OUTLOOK

We have presented a framework to quantify position-related information in measured UWB channels. As multipath propagation can be used to enhance accuracy and robustness of localization systems, we have shown how to evaluate the usefulness of different multipath components resulting from specular reflections. Results verify previous theoretical work on the Cramér-Rao bound for multipath-aided localization and provide important performance considerations for corresponding localization schemes. The beneficial influence of a high bandwidth has been shown and the influence of the geometry was emphasized. The presented methodology is potentially valuable in planning both localization systems and measurement campaigns. It is furthermore suitable to compare localization capability in different environments and frequency bands.

Future work most importantly has to address the path overlap problem, which was not considered here. Path overlap as one of the major issues for practical system requires additional algorithms to evaluate contributions and interference from multiple (virtual) anchors. Also, further parameterization and validation of the channel model has to be done, as the limited amount of closely spaced measurements did not allow for this here. Furthermore, coherent processing potentially provides additional gains in performance and has to be investigated.

REFERENCES