Advanced Signal Processing Seminar
Convex Optimization in Signal Processing

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Linear Programming and the Simplex method

A linear program (LP) is a (convex) optimization problem, where we aim to optimize a linear function subject to linear constraints:

$$\begin{aligned} \text{minimize} \quad & \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\,\mathbf{x} \leq \mathbf{b} \\ & \mathbf{C}\,\mathbf{x} = \mathbf{d} \end{aligned}$$

LPs are an important sub-class of convex problems, and their theory contributed strongly to the field of convex optimization. The Simplex algorithm for solving LPs, developed by Dantzig in the 1940's, was one of the first numerical optimization algorithms.

To do

- Review and present basic theory about LPs and the Simplex algorithm: How does the feasible set look like? When is an LP feasible? When is an LP unbounded? How does the solution of the Simplex method look like? What is a basic solution? How does the dual problem of an LP look like?
- Explain the Simplex algorithm in your presentation.
- Implement the Simplex algorithm (e.g. in Matlab). Your code does not need to run fast, but you have to implement it yourself. Also implement a method to find an initial feasible solution, e.g. the M-method or the two-phase method.
- Graphically (in 2D) demonstrate the Simplex algorithm, using a toy-example, e.g. the following:

$$\begin{aligned} \text{minimize} \quad & -x_1 - x_2 & \\ \text{subject to} \quad & -x_1 - x_2 & \leq -2 \\ & x_1 - x_2 & \leq 5 \\ & 3\,x_1 - x_2 & \leq 18 \\ & 3\,x_1 + x_2 & \leq 27 \\ & -x_1 + 5\,x_2 & \leq 25 \\ & -x_1 + x_2 & \leq 3 \\ & -4\,x_1 + x_2 & \leq 0 \end{aligned}$$

- Hand in your presentation slides and your code, which will be made available via our homepage.

Literature

- Boyd, Stephen and Vandenberghe, Lieven (2004). Convex Optimization. Cambridge University Press. http://www.stanford.edu/~boyd/cvxbook/
- Rainer Burkard, Lecture notes **Mathematische Optimierung**
http://www.opt.math.tu-graz.ac.at/~hatzl/Vorlesungen/MathoptSS11/Opt.pdf
- Juncheng Wei, Lecture notes **Linear Programming**
http://www.math.cuhk.edu.hk/~wei/LP11.html