Signal Processing and Speech Communication Lab.
Graz University of Technology

SV

Aufgabe 1

This homework has to be submitted via **e-mail** to the address `hw1.spsc@tugraz.at` **not later than 9.5.2018**. Let the **subject** of the e-mail be "YourMatrNo YourColleague'sMatrNo". The body of the e-mail should be empty (nobody will read it). A complete project consists of Matlab/Octave files (`*.m`) and a simulation protocol in PDF format. You have to **zip** all these files to a single file with name

`YourMatrNo_YourColleague's MatrNo.zip`

which has to be attached to the e-mail.

In addition to the email, you have to throw your printed (**paper format**) **simulation protocols and your analytic solutions** into our **mailbox at Inffeldgasse 16c**, ground floor, not later than 9.5.2018 (note that you cannot access the mailbox on weekends). For each problem, staple your solutions separately. Use the print-out of the respective problem assignment as the title page(s). Don't forget to fill in your **name(s)**, **matr. number(s)**, and **group number(s)**.

If you typeset your analytic solutions with LaTeX, you can get bonus points. If you submit a handwritten protocol, be aware that it should be clear, tidy and readable. Otherwise, you will lose points.

Name(s) Matr.Nr. **SV**

Analytic Problem 1.1 (8 points)

(a) [2 point(s)] Let

$$x[n] = \begin{cases} 0 & n < -2 \\ n & -2 \leq n \leq 3 \\ 0 & n > 3 \end{cases}$$

sketch the following signals

1. $y_1[n] = x[n+2]$
2. $y_2[n] = -x[-n-2]$
3. $y_3[n] = 2x[2n]$
4. $y_4[n] = \frac{1}{2}x[n] + \frac{1}{2}x[-n]$

(b) [2 point(s)] Analyze the following signals for periodicity. In case of periodicity, what is their period N_0?

1. $x_1[n] = e^{j\frac{2\pi}{N}nk} \quad \forall k, N, n \in \mathbb{Z}$
2. $x_2[n] = \sum_{k=0}^{4} e^{j\frac{2\pi k}{7}n} \quad \forall n \in \mathbb{Z}$
3. $x_3[n] = \cos(\frac{\pi-1}{\pi}n) \quad \forall n \in \mathbb{Z}$
4. $x_4[n] = \sin(3n) \quad \forall n \in \mathbb{Z}$

(c) [4 point(s)] Describe the following terms

- Linearity
- Time-invariance

Analyze the following systems for linearity and time-invariance where $x[n]$ and $y[n]$ denote the input and output signal.

1. $y_1[n] = x[n] + x[n-1]$
2. $y_2[n] = \sum_{i=n-2}^{n+4} x[-i]$
3. $y_3[n] = \cos(x[n]) + \delta[n]$
4. $y_4[n] = x[n-1]x[n+1]$

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Octave Problem 1.2 (17 points)

This sub assignment treats signal spaces. We use notation $()^*$ for conjugation, $()^T$ for transposed, $()^H$ for conjugate transposed (hermitian transposed), $\mathbf{x}$ denotes a column vector, $\mathbf{A}$ denotes a matrix. Throughout the following tasks we will explore the well known discrete Fourier transform in the context of signal spaces.

(a) [2 point(s)] Create a time vector $\mathbf{n} = [0, 1, \ldots, N-1]^T$ with length $N = 32$ and create the vectors $\mathbf{x}_1 = [x_1[0], x_1[1], \ldots, x_1[N-1]]^T$, $\mathbf{x}_2 = [x_2[0], x_2[1], \ldots, x_2[N-1]]^T$ where the signals $x_1[n]$ and $x_2[n]$ follow as

$$x_1[n] = 1 + \cos(\frac{2\pi}{16}n)$$

$$x_2[n] = j\delta[n-2]$$

for a range of $0 \leq n < N$ and imaginary unit $j = \sqrt{-1}$. Answer the following questions:

1. What is the dimension of $\mathbf{x}_1$, $\mathbf{x}_2$ and the product of $\mathbf{x}_1\mathbf{x}_2^H$?
2. Calculate the scalar products $y_1 = \mathbf{x}_1^H\mathbf{x}_2$ and $y_2 = \mathbf{x}_2^H\mathbf{x}_1$. What is their difference. What is the relation between y_1 and y_2?

(b) [2 point(s)] Plot the real-valued signal $x_1[n]$ using the plotting function `stem`. Use `help stem` to get information regarding the arguments of `stem`. Care for proper labeling of the axis using the commands `xlabel` and `ylabel` (one way to import the plots to your LaTeX document is exporting the figure as .eps file followed importing the .eps file in LaTeX using the command `\includegraphics[scale=\textwidth]{filename.eps}`).

Plot the complex-valued $x_2[n]$. Provide separate plots of the real and imaginary part using the commands `real` and `imag`, and plot the absolute value and the corresponding phase using `abs` and `angle`. Employ the command `subplot` in order to illustrate all four plots in one figure. Label the subplots with `title` and consider correct labeling of the axis!

(c) [2 point(s)] Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_L\}$ be a set of basis vectors, with

$$\mathbf{b}_l = [b_l[0], b_l[1], \ldots, b_l[N-1]]^T,$$

$L = N$ and

$$b_l[n] = \exp(j2\pi(l-1)n/N)$$

The basis $\mathcal{B}$ is an orthogonal basis. Let $\mathbf{c}_l = k\mathbf{b}_l$ be a scaled basis vector such that $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_L\}$ describes an orthonormal basis. What is the scaling value k as function of N (a mathematical proof is not required, just verify your findings for a few basis vectors (like $l = 2, 4, L$ for $N = 32$))?

(d) [2 point(s)] Plot $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$ and $\mathbf{c}_L$ using four subplots where each subplot consists of one basis vector and its real and imaginary parts are colored in blue and red! What can you observe? Are the vectors orthogonal?

(e) [3 point(s)] Create a function `A=getBasis(L)` which calculates the orthonormal basis $\mathcal{C}$ and returns the basis vectors stacked in matrix $\mathbf{A} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_L]^T$. The function is supposed to treat an arbitrary size of $L \in \mathbb{N}$. Do not use internal functions such as `dftmtx` or `fft`.

Verify your orthonormal basis by comparing $\mathbf{A}^{-1} = \mathbf{A}^H$. Create $\mathbf{A}$ with $L = 8$ and compute the squared error (using the command `norm`) between $\mathbf{A}^{-1}$ and $\mathbf{A}^H$ (should be zero, a small error below 10^{-15} can be reasoned by the software's limited numerical resolution).

(f) [3 point(s)] Let $\mathbf{x} \in \mathbb{C}^{N\times 1}$ be an arbitrary vector which can be expressed as scaled sum of basis vectors according to

$$\mathbf{x} = \sum_{l=1}^{L} \alpha_l \mathbf{c}_l \tag{1}$$

where the lth coefficient α_l can be calculated as scalar product $\alpha_l = \mathbf{c}_l^H \mathbf{x}$. Compute the coefficient vector $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_L]^T$ for the signal $\mathbf{x}_1$ using a single matrix vector multiplication. Plot $\boldsymbol{\alpha}$, what can you observe? For an arbitrary signal length N and number of basis vectors L, what is the minimum number of L required for describing $\mathbf{x}_1$? How many basis vectors L are required to describe $\mathbf{x}_2$?

(g) [3 point(s)] Use the computed coefficients $\boldsymbol{\alpha}$ and synthesize $\mathbf{x}_1$ by implementation of Equation (1) using a single matrix-vector multiplication. Then, copy the coefficients $\hat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}$ and set $\hat{\alpha}_1 = 0$. Recalculate (1) with $\hat{\boldsymbol{\alpha}}$, where the outcome is denoted as $\hat{\mathbf{x}}_1$. Plot both $\mathbf{x}_1$ and $\hat{\mathbf{x}}_1$ in one figure. What is the difference? How is it related to $\hat{\alpha}_1$ and $\mathbf{c}_1$?

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Analytic Problem 1.3 (9 points)

Let

$$x[n] = u[n+4] - u[n-1]$$
$$h[n] = \delta[n - n_0]$$

(a) [1 point(s)] Use the equation of the convolution sum to calculate $y_1[n] = (x * h)[n]$.

(b) [1 point(s)] Perform the convolution $y_2[n] = (x * x * h)[n]$ in time domain.

(c) [2 point(s)] Calculate the DTFT $Y_2(e^{j\theta})$ from $y_2[n]$ using the DTFT analysis equation. Factorize $Y_2(e^{j\theta}) = Y_{\mathrm{lp}}(e^{j\theta})Y_{\mathrm{r}}(e^{j\theta})$ where $Y_{\mathrm{lp}}(e^{j\theta}) = e^{j\theta\xi}$ contains the linear phase of $Y_2(e^{j\theta})$ with slope $\xi \in \mathbb{R}$, such that $Y_{\mathrm{r}}(e^{j\theta}) \in \mathbb{R}$. Calculate and draw the absolute value $|Y_{\mathrm{lp}}(e^{j\theta})|$ and phase $\angle Y_{\mathrm{lp}}(e^{j\theta})$ for a region of $-2\pi < \theta < 2\pi$.

(d) [2 point(s)] Calculate the DTFT $X(e^{j\theta})$ of $x[n]$ and $H(e^{j\theta})$ of $h[n]$. Factorize $X(e^{j\theta})$ to two parts: a linear phase part and a real-valued part (similar to **(c)**)!

(e) [2 point(s)] Recalculate $Y_2(e^{j\theta})$ in the Fourier domain using $X(e^{j\theta})$ and $H(e^{j\theta})$ (the property of $x[n] * y[n] \overset{\mathrm{DTFT}}{\longleftrightarrow} X(e^{j\theta})Y(e^{j\theta})$ might be helpful). Do you yield the same result as in **(c)**? Use the Dirichlet kernel property[1]

$$1 + 2\sum_{k=1}^{N} \cos(k\theta) = \frac{\sin \frac{2N+1}{2}\theta}{\sin \frac{\theta}{2}}$$

(f) [1 point(s)] Set n_0 such that $y_2[n]$ gets an even function $y_2[n] = y_2[-n]$ and draw $y_2[n]$.

[1] see http://www.math.unm.edu/∼crisp/courses/wavelets/fall09/chap4.pdf for more details