Signal Processing and Speech Communication Lab.
Graz University of Technology

AS

# Adaptive Systems—Homework Assignment 3

Name(s) Matr.No(s).

The analytical part of your homework (your calculation sheets) **as well as** the MATLAB simulation protocol have to be delivered as **hard copy** to our mailbox at Inffeldgasse 16c, ground floor, no later than **2018/2/21**. Use a printed version of **this entire document** as the title pages and fill in your **name(s) and matriculation number(s)**. Submitting your homework as a LaTeX document can earn you **up to 3 points!**
Your MATLAB programs (`*.m` files) and the simulation protocol (in pdf format!) have to be submitted via **e-mail** to the address `hw2.spsc@tugraz.at` no later than **2018/2/21**. The subject of the e-mail consists of the assignment number and your matriculation number(s) **"Assignment3, MatrNo1, MatrNo2"**. You have to zip (or tar) all your homework files to one single file with the name `Assignment3_MatrNo1_MatrNo2.zip`, e.g., `Assignment3_01312345_01312346.zip`, which has to be attached to the e-mail.
Please make sure that your approaches, procedures, and results are clearly presented. Justify your answers!

## Analytical Problem 3.1 (15 Points)—Predictive Encoding of an AR Process

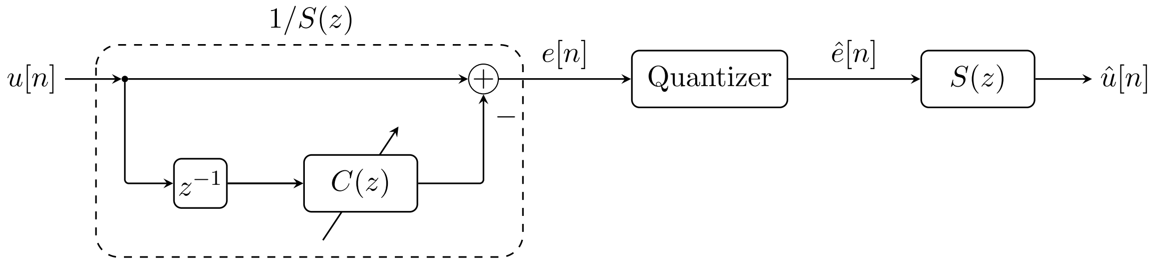



Figure 1: Open-loop prediction prior to quantization.

Let $u[n]$ be samples of an AR process with process generator difference equation

$$u[n] = w[n] + u[n-1] - \frac{1}{8}u[n-2]$$

where $w[n]$ are samples of white, zero-mean, Gaussian noise. The variance of the AR process is known to be $\sigma_u^2 = 1$.

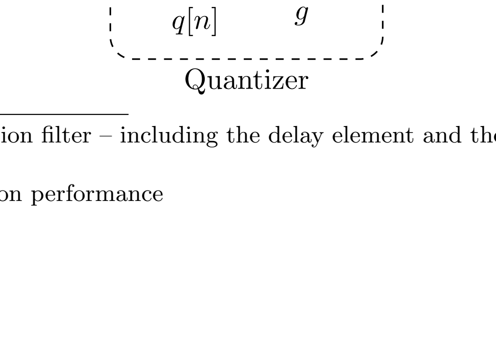
**(a)** Derive the first three samples of the autocorrelation sequence $r_{uu}[k]$, $k = 0, 1, 2$. Also, compute the variance of the white-noise input, $\sigma_w^2$, and the noise gain of the recursive process generator filter, $G_G = \frac{\sigma_u^2}{\sigma_w^2}$.

**(b)** For the above AR process, compute the MSE-optimal linear predictor of zeroth order[1], i.e., $C(z) = c_0$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

**(c)** For above AR process, compute the MSE-optimal linear predictor of first order, i.e., $C(z) = c_0 + c_1 z^{-1}$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

**(d)** Specify the transfer function of the synthesis filter $S(z)$ (i.e., the inverse of the prediction-error filter) for both $N = 1$ and $N = 2$. Assume that no quantization was performed. Calculate the noise gain $G_S$ of $S(z)$. Can you use the noise gain of $S(z)$ to compute the variance of $\hat{u}[n]$?

**(e)** We apply the following model for an ideal[2] quantizer for Gaussian sources:




[1] Note that the overall prediction filter – including the delay element and the direct path depicted in Fig. 1 – is a first-order filter.

[2] according to its rate-distortion performance

Here, the quantization noise $q[n]$ is white, uncorrelated with the quantizer input, and has a variance given as

$$\sigma_q^2 = \frac{\sigma_e^2}{2^{2R}-1}$$

where $R = 1$ is the bit rate. The gain inside the quantizer is given by

$$g = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_q^2}.$$

For both $N = 1$ and $N = 2$ determine the variance $\sigma_r^2$ of the reconstruction error $r[n] = \hat{u}[n] - u[n]$ at the synthesis filter output.

**(f)** Compare above reconstruction error with the distortion achieved by direct encoding of $u[n]$ with 1 bit/sample (i.e., without prediction). Is there any gain by using prediction?

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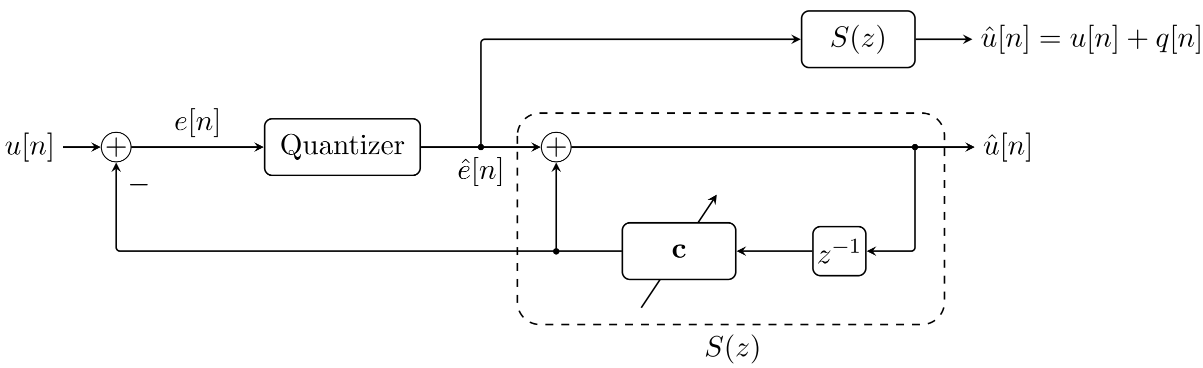
## Analytical Problem 3.2 (5 Points)—Closed-Loop Prediction



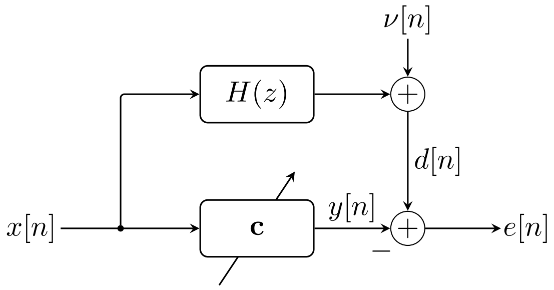

In the previous problem we examined open-loop prediction. In this problem we will take a look at closed loop prediction. Model again the quantizer as an uncorrelated, white noise source with variance $\sigma_q^2 = \gamma\sigma_e^2$, where $\gamma$ is a constant (that is related to the resolution) and where $\sigma_e^2$ is the variance of the signal at the input of the quantizer.

$$\hat{e}[n] = e[n] + q[n].$$

**(a)** Show that the so-called "error identity" holds: $\hat{u}[n] = u[n] + q[n]$. In other words, the quantization error appears directly at the output.

**(b)** Argue that, as a consequence of the error identity, closed-loop prediction <u>can</u> reduce the reconstruction error variance of the digital transmission system.

## MATLAB Problem 3.3 (12 Points)—IIR System Identification




We want to identify the following second-order (IIR) all-pole filter:

$$H(z) = \frac{1}{1 + 0.8z^{-1} + 0.8z^{-2}}$$

To this end, we use the LMS algorithm to adapt an 11th order ($N = 12$) FIR filter with coefficient vector $\mathbf{c}$. Let $\nu[n]$ be a zero-mean white noise process with variance $\sigma_\nu^2 = 0.01$. $\nu[n]$ and $x[n]$ are uncorrelated in each of the following cases, which you should consider:

1. $x[n]$ is a zero-mean white noise process with unit variance.
2. $x[n]$ is the output of a filter $G(z) = 1 + z^{-1}$, with a zero-mean, variance one-half, white noise process as its input.
3. $x[n]$ is the output of a filter $G(z) = 1 - z^{-1}$, with a zero-mean, variance one-half, white noise process as its input.

For each of these input signals, perform the following tasks:

**(a)** Show that the variance of each of these signals is $\sigma_x^2 = 1$.

**(b)** Compute the autocorrelation matrix $\mathbf{R}_{xx}$. You don't need to write down the $12 \times 12$ matrix, it suffices to write down, e.g., the upper left $4 \times 4$ block.

**(c)** For a general step size $\mu$, estimate the convergence time constants of the slowest and the fastest modes, $\tau_{\max}$ and $\tau_{\min}$. **Hint:** Use MATLAB/Octave to compute the eigenvalues of $\hat{\mathbf{R}}_{xx}$. For which signal would a gradient search algorithm (or an LMS) converge the fastest?

**(d)** Use your implementation of the LMS to find the filter coefficient vector $\mathbf{c}[n]$. Let $n$ be large enough such that the LMS has converged. Plot the impulse response $\mathbf{c}[n]$ and compare it to the impulse response of the IIR system, $h[n]$, which you can obtain using the command `impz`. What do you observe?

**(e)** Now use `freqz` to compare the different solutions for $\mathbf{c}[n]$ with the magnitude response of the IIR filter. Give an explanation for the observed behavior!

## MATLAB Problem 3.4 (7 Points)—BONUS: Periodic Interference Cancelation

On the course web page you will find a wave file (8 kHz sampling rate) that contains speech with interfering DTMF signals. Implement an interference cancelation system of your choice in MATLAB that removes/attenuates the interference. This can for example be an LMS-based predictor, an adaptive/non-adaptive filter bank or a hybrid system using both adaptive and non-adaptive filters.

The only requirement the system has to satisfy is a maximum input-to-output delay of 25 ms. You can use any delay $\Delta \leq 25$ ms, but you have to tell us the exact value for $\Delta$ of your implementation in samples.

For evaluation you are provided with the clean speech signal $s[n]$ (which, of course, you are NOT allowed to use for interference cancelation). As a performance metric of your system you shall use the SNR computed as

$$\mathrm{SNR} = 10 \log_{10} \frac{\sigma_s^2}{\sigma_r^2}$$

where $r[n]$ is any error on the output signal $\hat{s}[n]$ of your system ($r[n] = s[n - \Delta] - \hat{s}[n]$).