Signal Processing and Speech Communication Lab.
Graz University of Technology

AS

Adaptive Systems—Homework Assignment 3

v1.0

Name(s) Matr.No(s).

The analytical part of your homework (your calculation sheets) **as well as** the MATLAB simulation protocol have to be delivered either as **hard copy** to our mailbox at Inffeldgasse 16c, ground floor or can be **uploaded** to the TeachCenter, in both cases no later than **2019/2/21**. Use **this page** as title page, filling in your **name(s) and matriculation number(s)**. Submitting your homework as a LaTeX document can earn you **up to 3 points!**
If you hand in a hard copy, your Matlab/Octave programs (`*.m` files) and the simulation protocol (in pdf format!) have to be submitted via **e-mail** to the address `hw2.spsc@tugraz.at` no later than **2019/2/21**. The subject of the e-mail consists of the assignment number and your matriculation number(s) **"Assignment3, MatrNo1, MatrNo2"**. You have to zip (or tar) all your homework files to one single file with the name `Assignment3_MatrNo1_MatrNo2.zip`, e.g., `Assignment3_01312345_01312346.zip`, which has to be attached to the e-mail. Otherwise upload your Matlab/Octave programs to the TeachCenter no later than **2019/2/21**.
Please make sure that your approaches, procedures, and results are clearly presented. Justify your answers!

Analytical Problem 3.1 (13 Points)—Linear Prediction of an AR Process

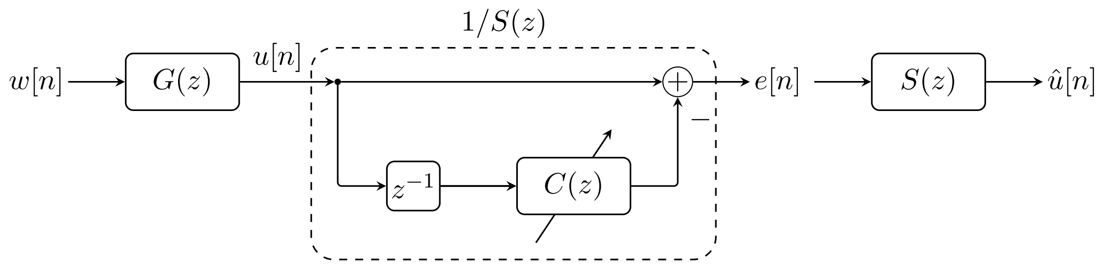


Figure 1: Adaptive linear predictor.

Let $u[n]$ be samples of an AR process with process generator difference equation

$$u[n] = w[n] + \frac{2}{3}u[n-1] - \frac{1}{3}u[n-2]$$

where $w[n]$ are samples of white, zero-mean, Gaussian noise. The variance of the AR process is known to be $\sigma_u^2 = 1$.

(a) Derive the first three samples of the autocorrelation sequence $r_{uu}[k]$, $k = 0, 1, 2$. Also, compute the variance of the white-noise input, σ_w^2, and the noise gain of the recursive process generator filter, $G_G = \frac{\sigma_u^2}{\sigma_w^2}$. Also explicitly write down $G(z)$.

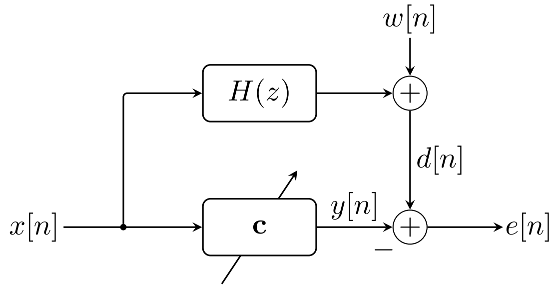
(b) For the above AR process, compute the MSE-optimal linear predictor of zeroth order[1], i.e., $C(z) = c_0$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

(c) For the above AR process, compute the MSE-optimal linear predictor of first order, i.e., $C(z) = c_0 + c_1 z^{-1}$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

(d) Specify the transfer function of the synthesis filter $S(z)$ (i.e., the inverse of the prediction-error filter) for both $N = 1$ and $N = 2$ and compare it to G(z). Calculate the noise gain G_S of $S(z)$. Can you use the noise gain of $S(z)$ to compute the variance of $\hat{u}[n]$?

[1]Note that the overall prediction filter – including the delay element and the direct path depicted in Fig. 1 – is a first-order filter.

MATLAB Problem 3.2 (16 Points)—IIR System Identification



We want to identify the following second-order (IIR) all-pole filter:

$$H(z) = \frac{1}{1 + 0.8z^{-1} + 0.8z^{-2}}$$

To this end, we use the LMS algorithm to adapt an 11th order ($N = 12$) FIR filter with coefficient vector $\mathbf{c}$. Let $w[n]$ be a zero-mean white noise process with variance $\sigma_w^2 = 0.01$. $w[n]$ and $x[n]$ are uncorrelated in each of the following cases, which you should consider:

1. $x[n]$ is a zero-mean white noise process with unit variance.
2. $x[n]$ is the output of a filter $G(z) = 1 + z^{-1}$, with a zero-mean, variance one-half, white noise process as its input.
3. $x[n]$ is the output of a filter $G(z) = 1 - z^{-1}$, with a zero-mean, variance one-half, white noise process as its input.

For each of these input signals, perform the following tasks:

(a) Show that the variance of each of these signals is $\sigma_x^2 = 1$.

(b) Compute the autocorrelation matrix $\mathbf{R}_{xx}$. You don't need to write down the 12×12 matrix, it suffices to write down, e.g., the upper left 4×4 block.

(c) Compute numerical estimates $\hat{\mathbf{R}}_{xx}$ of the autocorrelation matrix $\mathbf{R}_{xx}$ (i) via taking the time average over $\mathbf{x}[n]\mathbf{x}[n]^T$ and (ii) via taking the ensemble average. Compare these results with your autocorrelation matrix from **(b)**. Show how the number of ensembles/time samples used for averaging influences the estimate of $\mathbf{R}_{xx}$.

(d) For a general step size μ, estimate the convergence time constants of the slowest and the fastest modes, $\tau_{\max}$ and $\tau_{\min}$. **Hint:** Use Matlab/Octave to compute the eigenvalues of $\hat{\mathbf{R}}_{xx}$. For which signal would a gradient search algorithm (or an LMS) converge the fastest?

(e) Use your implementation of the LMS to find the filter coefficient vector $\mathbf{c}[n]$. Let n be large enough such that the LMS has converged. Plot the impulse response $\mathbf{c}[n]$ and compare it to the impulse response of the IIR system, $h[n]$, which you can obtain using the command `impz`. What do you observe?

(f) Now use `freqz` to compare the different solutions for $\mathbf{c}[n]$ with the magnitude response of the IIR filter. Give an explanation for the observed behavior! Also examine the influence of the number of filter coefficients, e.g. also compare your results for $N = 6$ and $N = 12$.
Hint: Think about the influence of $G(z)$ on its input.

MATLAB Problem 3.3 (10 Points)—Bonus Challenge: Channel Equalization

With the small receiver of your farm station you were able to record a message[2] from an unknown source in outer space. Unfortunately it was distorted quite badly so you were not able to decode it properly (see file `receivedSignal.mat`). Some days later, when talking to your neighbors, you find out that they actually were able to record and decode the beginning by pure coincidence, but did not record the rest. As you are intrigued what the message might contain, they give you their recorded piece (see file `trainingSequence.mat`).
Try to decode the message that was transmitted over an unknown channel. You were already able to find out the following properties of the transmitted message:

- The channel is a time-varying FIR filter; you can assume that at least at the beginning of the message the largest tap is positive and at the beginning of the impulse response. Moreover, the channel order will be less than 10 (you are far from any mountains and there are not many reflections in outer space).
- The transmitter seems to have moved away from you, so the SNR decreases over time.
- You know that every message is transmitted as an ASCII code, where ones are transmitted as “1” and zeros as “-1”. You can convert eight integers (zeros and ones) to a character by using `char(int2str())`.
- The message starts with the message fragment that you got from your neighbors:
  “`CR90 corvette Tantive IV requests a transmission to Tatooine: `”
  (see file `trainingSequence.mat`).

The team that decodes most of the message[a] (i.e., the team with the lowest bit error rate) wins a prize!

[a]We will perform another test with a different message.

[2]Names, setting, part of the transmitted message are the intellectual property of Lucasfilm. Copyright lies with the associated labels.