Adaptive Systems—Homework Assignment 3

Name(s)  Matr.No(s).

The analytical part of your homework (your calculation sheets) as well as the MATLAB simulation protocol have to be delivered as hard copy to our mailbox at Inffeldgasse 16c, ground floor, no later than 2018/2/21. Use a printed version of this entire document as the title pages and fill in your name(s) and matriculation number(s). Submitting your homework as a LaTeX document can earn you up to 3 points!

Your MATLAB programs (*.m files) and the simulation protocol (in pdf format!) have to be submitted via e-mail to the address hw2.spsc@tugraz.at no later than 2018/2/21. The subject of the e-mail consists of the assignment number and your matriculation number(s) “Assignment3, MatrNo1, MatrNo2”. You have to zip (or tar) all your homework files to one single file with the name Assignment3_MatrNo1_MatrNo2.zip, e.g., Assignment3_01312345_01312346.zip, which has to be attached to the e-mail.

Please make sure that your approaches, procedures, and results are clearly presented. Justify your answers!
Analytical Problem 3.1 (15 Points)—Predictive Encoding of an AR Process

Let \( u[n] \) be samples of an AR process with process generator difference equation

\[
u[n] = w[n] + u[n-1] - \frac{1}{8} u[n-2]
\]

where \( w[n] \) are samples of white, zero-mean, Gaussian noise. The variance of the AR process is known to be \( \sigma_u^2 = 1 \).

(a) Derive the first three samples of the autocorrelation sequence \( r_{uu}[k], k = 0, 1, 2 \). Also, compute the variance of the white-noise input, \( \sigma_w^2 \), and the noise gain of the recursive process generator filter, \( G_G = \frac{\sigma_u^2}{\sigma_w^2} \).

(b) For the above AR process, compute the MSE-optimal linear predictor of zeroth order, i.e., \( C(z) = c_0 \). Also, compute the prediction gain \( G_C = \frac{\sigma_u^2}{\sigma_e^2} \). Is the used predictor order optimal for the given source?

(c) For the above AR process, compute the MSE-optimal linear predictor of first order, i.e., \( C(z) = c_0 + c_1 z^{-1} \). Also, compute the prediction gain \( G_C = \frac{\sigma_u^2}{\sigma_e^2} \). Is the used predictor order optimal for the given source?

(d) Specify the transfer function of the synthesis filter \( S(z) \) (i.e., the inverse of the prediction-error filter) for both \( N = 1 \) and \( N = 2 \). Assume that no quantization was performed. Calculate the noise gain \( G_S \) of \( S(z) \). Can you use the noise gain of \( S(z) \) to compute the variance of \( \hat{u}[n] \)?

(e) We apply the following model for an ideal quantizer for Gaussian sources:

\[ e[n] \rightarrow \text{Quantizer} \rightarrow \hat{e}[n] \]

\[ q[n] \rightarrow g \rightarrow \hat{e}[n] \]

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1 Note that the overall prediction filter — including the delay element and the direct path depicted in Fig. 1 — is a first-order filter.

2 According to its rate-distortion performance.
Here, the quantization noise $q[n]$ is white, uncorrelated with the quantizer input, and has a variance given as

$$\sigma_q^2 = \frac{\sigma_e^2}{2^{2N} - 1}$$

where $R = 1$ is the bit rate. The gain inside the quantizer is given by

$$g = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_q^2}.$$ 

For both $N = 1$ and $N = 2$ determine the variance $\sigma_r^2$ of the reconstruction error $r[n] = \hat{u}[n] - u[n]$ at the synthesis filter output.

(f) Compare above reconstruction error with the distortion achieved by direct encoding of $u[n]$ with 1 bit/sample (i.e., without prediction). Is there any gain by using prediction?

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**Analytical Problem 3.2 (5 Points)—Closed-Loop Prediction**

\[ u[n] + e[n] \xrightarrow{\text{Quantizer}} \hat{e}[n] + q[n] \xrightarrow{\text{Gain} c} \hat{u}[n] \]

\[ \hat{u}[n] \xrightarrow{S(z)} \hat{u}[n] = u[n] + q[n] \]

In the previous problem we examined open-loop prediction. In this problem we will take a look at closed-loop prediction. Model again the quantizer as an uncorrelated, white noise source with variance $\sigma_q^2 = \gamma \sigma_e^2$, where $\gamma$ is a constant (that is related to the resolution) and where $\sigma_e^2$ is the variance of the signal at the input of the quantizer.

\[ \hat{e}[n] = e[n] + q[n]. \]

(a) Show that the so-called “error identity” holds: $\hat{u}[n] = u[n] + q[n]$. In other words, the quantization error appears directly at the output.

(b) Argue that, as a consequence of the error identity, closed-loop prediction can reduce the reconstruction error variance of the digital transmission system.
MATLAB Problem 3.3 (12 Points)—IIR System Identification

We want to identify the following second-order (IIR) all-pole filter:

\[ H(z) = \frac{1}{1 + 0.8z^{-1} + 0.8z^{-2}} \]

To this end, we use the LMS algorithm to adapt an 11th order \((N = 12)\) FIR filter with coefficient vector \(c\). Let \(\nu[n]\) be a zero-mean white noise process with variance \(\sigma^2_{\nu} = 0.01\). \(\nu[n]\) and \(x[n]\) are uncorrelated in each of the following cases, which you should consider:

1. \(x[n]\) is a zero-mean white noise process with unit variance.
2. \(x[n]\) is the output of a filter \(G(z) = 1 + z^{-1}\), with a zero-mean, variance one-half, white noise process as its input.
3. \(x[n]\) is the output of a filter \(G(z) = 1 - z^{-1}\), with a zero-mean, variance one-half, white noise process as its input.

For each of these input signals, perform the following tasks:

**(a)** Show that the variance of each of these signals is \(\sigma^2_x = 1\).

**(b)** Compute the autocorrelation matrix \(R_{xx}\). You don’t need to write down the 12×12 matrix, it suffices to write down, e.g., the upper left 4×4 block.

**(c)** For a general step size \(\mu\), estimate the convergence time constants of the slowest and the fastest modes, \(\tau_{\text{max}}\) and \(\tau_{\text{min}}\). **Hint:** Use MATLAB/Octave to compute the eigenvalues of \(\hat{R}_{xx}\). For which signal would a gradient search algorithm (or an LMS) converge the fastest?

**(d)** Use your implementation of the LMS to find the filter coefficient vector \(c[n]\). Let \(n\) be large enough such that the LMS has converged. Plot the impulse response \(c[n]\) and compare it to the impulse response of the IIR system, \(h[n]\), which you can obtain using the command `impz`. What do you observe?

**(e)** Now use `freqz` to compare the different solutions for \(c[n]\) with the magnitude response of the IIR filter. Give an explanation for the observed behavior!
MATLAB Problem 3.4 (7 Points)—BONUS: Periodic Interference Cancelation

On the course web page you will find a wave file (8 kHz sampling rate) that contains speech with interfering DTMF signals. Implement an interference cancelation system of your choice in MATLAB that removes/attenuates the interference. This can for example be an LMS-based predictor, an adaptive/non-adaptive filter bank or a hybrid system using both adaptive and non-adaptive filters.

The only requirement the system has to satisfy is a maximum input-to-output delay of 25 ms. You can use any delay $\Delta \leq 25$ ms, but you have to tell us the exact value for $\Delta$ of your implementation in samples.

For evaluation you are provided with the clean speech signal $s[n]$ (which, of course, you are NOT allowed to use for interference cancelation). As a performance metric of your system you shall use the SNR computed as

$$SNR = 10 \log_{10} \frac{\sigma_s^2}{\sigma_r^2}$$

where $r[n]$ is any error on the output signal $\hat{s}[n]$ of your system ($r[n] = s[n - \Delta] - \hat{s}[n]$).