

Fundamentals of Digital Communications Formulary

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1 Definition of Special Functions

Rectangle Function:

$$\text{rect}(x, X) = \begin{cases} 1; & |x| \leq X \\ 0; & |x| > X \end{cases}$$

Unit Step Function:

$$u(x) = \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases}$$

Sinc Function:

$$\text{sinc}(x) = \begin{cases} 1; & x = 0 \\ \frac{\sin(\pi x)}{\pi x}; & x \neq 0 \end{cases}$$

2 Fourier Series

Fourier Series:

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk(2\pi/T)t}$$

Fundamental Frequency:

$$f_0 = \frac{1}{T}; \quad \omega_0 = \frac{2\pi}{T}$$

Fourier Coefficients:

$$c_k = \frac{1}{T} \int_{t=t_0}^{t_0+T} x(t) e^{-jk(2\pi/T)t} dt$$

3 Fourier Transform

Fourier Transform:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

Inverse Fourier Transform:

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi ft} df$$

Even and Odd Parts of Functions:

$$x_{\text{even}}(t) = \frac{1}{2} (x(t) + x^*(-t))$$

$$x_{\text{odd}}(t) = \frac{1}{2} (x(t) - x^*(-t))$$

$$X_{\text{even}}(f) = \frac{1}{2} (X(f) + X^*(-f))$$

$$X_{\text{odd}}(f) = \frac{1}{2} (X(f) - X^*(-f))$$

Table 1: Symmetry of the Fourier Transforms

$x(t)$	\leftrightarrow	$X(f)$
$x(-t)$	\leftrightarrow	$X(-f)$
$x^*(t)$	\leftrightarrow	$X^*(-f)$
$x^*(-t)$	\leftrightarrow	$X^*(f)$
$\Re\{x(t)\}$	\leftrightarrow	$X_{\text{even}}(f)$
$j\Im\{x(t)\}$	\leftrightarrow	$X_{\text{odd}}(f)$
$x_{\text{even}}(t)$	\leftrightarrow	$\Re\{X(f)\}$
$x_{\text{odd}}(t)$	\leftrightarrow	$j\Im\{X(f)\}$

Table 2: Properties of the Fourier Transform

$ax(t) + by(t)$	\leftrightarrow	$aX(f) + bY(f)$
$x(at)$	\leftrightarrow	$\frac{1}{ a }X\left(\frac{f}{a}\right)$
$x(t) * y(t)$	\leftrightarrow	$X(f)Y(f)$
$x(t)y(t)$	\leftrightarrow	$X(f) * Y(f)$
$x(t - \tau)$	\leftrightarrow	$X(f)e^{-j2\pi f\tau}$
$e^{j2\pi f_0 t}x(t)$	\leftrightarrow	$X(f - f_0)$
$\frac{d^m x(t)}{dt^m}$	\leftrightarrow	$(j2\pi f)^m X(f)$
$\left(\frac{-jt}{2\pi}\right)^m x(t)$	\leftrightarrow	$\frac{d^m X(f)}{df^m}$
$\int_{-\infty}^t x(\tau)d\tau$	\leftrightarrow	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}\delta(f) \int_{-\infty}^{\infty} x(\tau)d\tau$
$x(t) \cos(2\pi f_0 t)$	\leftrightarrow	$\frac{1}{2}(X(f - f_0) + X(f + f_0))$
$X(t)$	\leftrightarrow	$x(-f)$

Table 3: Fourier Transform Pairs

$e^{j2\pi f_0 t}$	\leftrightarrow	$\delta(f - f_0)$
$\delta(t - T)$	\leftrightarrow	$e^{-j2\pi fT}$
$\cos(2\pi f_0 t)$	\leftrightarrow	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
$\sin(2\pi f_0 t)$	\leftrightarrow	$\frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$
$\text{sinc}(Ft)$	\leftrightarrow	$\frac{1}{F}\text{rect}(f, F/2)$
$\text{rect}(t, T/2)$	\leftrightarrow	$T\text{sinc}(fT)$
$e^{-\alpha t}u(t)$	\leftrightarrow	$\frac{1}{j2\pi f + \alpha}; \Re\{\alpha\} > 0$
$u(t)$	\leftrightarrow	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$
$\sum_{k=-\infty}^{+\infty} \delta(t - kT)$	\leftrightarrow	$\frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta(f - \frac{1}{T}m)$
$\frac{1}{jt}$	\leftrightarrow	$-\pi \text{sgn}(f)$

4 Q-Function

$$Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

$$Q(-x) = 1 - Q(x)$$

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\operatorname{erfc}(x) = 2Q(\sqrt{2}x)$$

Table 4: Q-Function Table; $Q(x)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002