Institut für Signalverarbeitung und Spachkommunikation, Technische Universität Graz

Exam for Fundamentals of Digital Communications (2VO) on 30-6-2017

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Duration: 3 Stunden
Permitted material: Table of Fourier transform properties, tabulated Q-function (both are provided), pocket calulator. **The assignment sheets and provided tables must be returned in the end of the exam!**

Theory 1 (30 Points)

(a) (10 Points) Why is a Gaussian (normal) random distribution of such great importance? Why is noise in digital communication systems distributed according to a Gaussian probability density function (PDF)? Sketch the PDF and the cumulative distribution function (CDF) of a Gaussian random variable.
(b) (10 Points) Discuss the main components of an optimal receiver for digital waveforms. Draw the block diagram. Explain the purpose of each essential block. Why is the receiver said to be optimal (for the AWGN channel)? What criteria are used to optimize the various blocks?
(c) (10 Points) Linear operators: assume an operator is defined as $\mathbf{A} : \mathcal{H}_0 \to \mathcal{H}_1$. Which two properties have to hold that $\mathbf{A}$ is a *linear* operator? Explain the range and the nullspace of a linear operator.

Problem 1 (15 Points)

A random variable X is defined by the CDF

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}x, & 0 \leq x < 1 \\ \frac{1}{2}x - \frac{1}{4}, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

(a) Sketch $F_X(x)$ and the PDF $f_X(x)$.
(b) What is the probability that $0 < X < 1$?
(c) What is the probability that $1.5 \leq X < 2$?
(d) What is the probability that $X = 1$? What is the probability that $X = 2$?

Problem 2 (30 Points)

In a binary digital communication system, two signal waveforms are transmitted, which are defined by the signal vectors

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{\mathcal{E}_b}/2 \\ \sqrt{3\mathcal{E}_b}/2 \end{bmatrix} \quad \text{and} \quad \mathbf{s}_2 = \begin{bmatrix} \sqrt{\mathcal{E}_b}/2 \\ -\sqrt{3\mathcal{E}_b}/2 \end{bmatrix},$$

in a signal space given by the basis functions

$$\psi_1(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases} \quad \text{and} \quad \psi_2(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1/2 \\ -1 & \text{for } 1/2 < t \le 1 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Sketch the two signal waveforms.
(b) Compute the energies of these signals, the angle between the signal vectors, and determine the distance between the signal vectors.
(c) Sketch the matched-filter demodulator for these signals, determine the impulse responses of the matched filter(s), and explain, why this is an optimal receiver structure for an AWGN channel.
(d) The signals are transmitted at equal a-priori probabilities over an AWGN channel with noise power spectral density $S_n(f) = N_0/2$. Determine the decision rule for optimal detection (obtaining minimum error probability).
(e) Compute the error probability for a signal-to-noise ratio per bit of $\mathcal{E}_b/N_0 = 8 = 9$ dB.

Problem 3 (25 Points)

Sketch the vectorial signal represenation and compare the minimum distance at a fixed bit energy $\mathcal{E}_b$, for the following digital modulation schemes.
(a) binary phase shift keying (BPSK)
(b) on-off-keying (OOK)
(c) binary orthogonal signaling
(d) quarternary phase shift keying (QPSK)
(e) symmetric 4-ary pulse amplitude modulation (4-PAM)

Which modulation scheme is best in terms of rate performance and spectral efficiency for transmission over an AWGN channel and why?