Institut für Signalverarbeitung und Spachkommunikation, Technische Universität Graz

Exam for Fundamentals of Digital Communications (2VO) on 10-3-2017

Name MatrNr. StudKennz.

Duration: 3 Stunden
Permitted material: Table of Fourier transform properties, tabulated Q-function (both are provided), pocket calulator. **The assignment sheets and provided tables must be returned in the end of the exam!**

Theory 1 (30 Points)

(a) How is an orthonormal basis defined for an N dimensional signal space? How could you derive such a basis (which algorithm)? How does this algorithm work?
(b) What problem is solved by a matched filter? How is it obtained? What performance parameter is optimized by the matched filter? How many matched filters do you need for an N dimensional signal space?
(c) What is a bandpass signal? What is the equivalent lowpass representation of a bandpass signal? How can it be interpreted, considering complex-number arithmetic? Define a mathematical relation between the two signal representations and illustrate them in the frequency domain.

Problem 1 (15 Points)

In a certain city three car brands, A, B, C have 20 %, 30 %, and 50 % of the market share, respectively. The probability that a car needs major repair during its first year of purchase for the three brands is 5%, 10%, and 15%, respectively.
(a) What is the probability that a car in this city needs major repair during its first year of purchase?
(b) If a car in this city needs major repair during its first year of a purchase, what is the probability that it is made by manufacturer A?

Problem 2 (30 Points)

Two signals $s_1(t)$ and $s_2(t)$ are defined as

$$s_1(t) = \begin{cases} At & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$s_2(t) = \begin{cases} B & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the norm of $s_1(t)$ and $s_2(t)$ in dependence of A and B, respectively.
(b) Find the inner product of these two signals in a linear space. What is the angle between the two signals?
(c) Find an orthonormal basis for the signal space $\mathcal{S}$ which is spanned by $s_1(t)$ and $s_2(t)$.
(d) The signals $s_1(t)$ and $s_2(t)$ should have the same signal energy $\mathcal{E}_s$. Determine A and B as a function of $\mathcal{E}_s$ to fulfill this constraint.
(e) The signals $s_1(t)$ and $s_2(t)$ (with symbol energy $\mathcal{E}_s$) are signals in a M-PSK modulation scheme (consider a uniformly spaced constellation). Determine the number of symbols M and sketch the constellation diagram.
(f) Determine the number of nearest neighbours $\overline{N}_e$ of a symbol and the distance between these neighboring symbols as a function of $\mathcal{E}_s$.

Problem 3 (25 Points)

A binary antipodal transmission system uses the following two signals for the transmission of two bits.

$$s_1(t) = -s_2(t) = \begin{cases} \sqrt{\mathcal{E}_b} & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

The signals are transmitted over an AWGN channel with double-sided noise power spectral density of $N_0/2$. The two signals have prior probabilities $p_1 = 10$ % and $p_2 = 1 - p_1$.
(a) Determine the structure of the optimal receiver (matched filter type or correlator type).
(b) Give the equations for the likelihood functions and illustrate them.
(c) Determine metrics for an optimum decision according to the maximum a-posteriori (MAP) criterion.
(d) Determine a threshold value for the MAP decision.
(e) Compute the error probability for $E_b/N_0 = 2 = 3$ dB.