Institut für Signalverarbeitung und Spachkommunikation, Technische Universität Graz

Exam for Fundamentals of Digital Communications (2VO) on 1-12-2015

Name MatrNr. StudKennz.

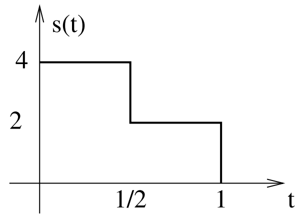
Duration: 3 Stunden
Permitted material: Table of Fourier transform properties, tabulated Q-function (both are provided), and pocket calulator. **The assignment sheets and provided tables must be returned in the end of the exam!**

Theory 1 (30 Points)

(a) Why is a Gaussian (normal) random distribution of such great importance? Why is noise in digital communication systems distributed according to a Gaussian probability density function (PDF)? Sketch the PDF and the cumulative distribution function (CDF) of a Gaussian random variable.
(b) What is a bandpass signal? What is the equivalent lowpass representation of a bandpass signal? How can it be interpreted, considering complex-number arithmetic? Define a mathematical relation between the two signal representations and illustrate them in the frequency domain.
(c) Discuss the trade-off between bandwidth, transmission rate, and signal-to-noise ratio per bit (E_b/N_0) to reach a certain symbol error rate (SER). Discuss the cases that (i) the bandwidth of a system is limited and (ii) its power is limited. What is needed to ensure reliable communication? (iii) For a given modulation scheme, how is the bandwidth related to the transmission rate?

Problem 1 (20 Points)

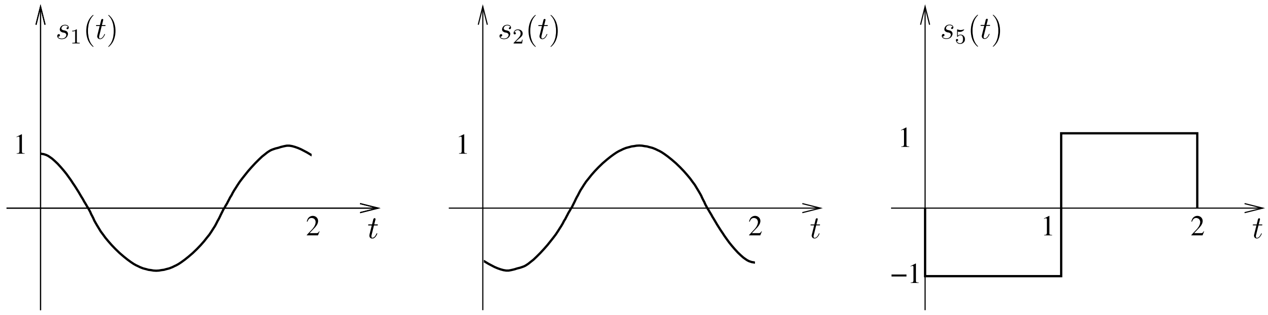
Matched Filter. The pulse shape depicted below is used for data transmission.



(a) Sketch the impulse response of a receiver filter matched to this pulse shape.
(b) Determine the output waveform of this filter for the transmitted pulse shape. (Convolution integral!)
(c) Indicate the sampling time instance that should be used for the signal detection and compute the filter output value at this time instance.

Problem 2 (25 Points)

Given three signals $s_1(t)$ and $s_2(t)$ and $s_5(t)$:



(a) Find the norm of $s_1(t)$, $s_2(t)$, where

$$s_1(t) = \begin{cases} \cos(\pi t + \pi/6) & 0 \le t \le 2 \\ 0 & \text{elsewhere} \end{cases}, \quad s_2(t) = \begin{cases} -\cos(\pi t - \pi/6) & 0 \le t \le 2 \\ 0 & \text{elsewhere} \end{cases}.$$

(b) Find the inner product of these two signals in a linear space. What is the angle between the two signals?
(c) Find the norm of the signal $s_3(t) = s_1(t) + s_2(t)$. Sketch the signal $s_3(t)$.
(d) Find a signal $s_4(t)$ that is *in* the subspace $\mathcal{S}$ spanned by $s_1(t)$ and $s_2(t)$ and is orthogonal to $s_3(t)$.
(e) Sketch the signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ in a vector space. How many dimensions do these signals span?
(f) Find and sketch the signal $s_6(t)$ that lies in the subspace $\mathcal{S}$ spanned by $s_1(t)$ and $s_2(t)$ and that is closest to $s_5(t)$. Which signals $\in \mathcal{S}$ do you choose for determining this *projection*? Argue this choice! What properties need to be fulfilled?
(g) Compute and sketch the projection error, resulting from projecting $s_5(t)$ onto the subspace $\mathcal{S}$ spanned by $s_1(t)$ and $s_2(t)$.

Problem 3 (25 Points)

A binary transmission system is defined by two Gaussian likelihood functions $f(r|s_1)$ and $f(r|s_2)$, with mean values $s_1 = 0$ and $s_2 = 1$, respectively, and variance $\sigma_n^2 = 1/4$. The symbol s_1 is sent with a-priori probability of $P(s_1) = 0.9$.
(a) Give the equations for the likelihood functions and illustrate them.
(b) Determine metrics for an optimum decision according to the maximum a-posteriori (MAP) criterion. Write down the determined metrics as a function of r.
(c) Determine a threshold value for the MAP decision and indicate it in the figure.
(d) Determine the error probability for the MAP decision.
(e) Compute the mean energy per bit for the transmitted symbols $s \in \{s_1, s_2\}$.