Institut für Signalverarbeitung und Spachkommunikation, Technische Universität Graz

Exam for Fundamentals of Digital Communications (2VO) on 29-5-2015

Name MatrNr. StudKennz.

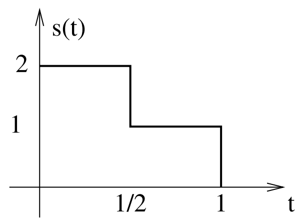
Duration: 3 Stunden
Permitted material: Table of Fourier transform properties, tabulated Q-function (both are provided), and pocket calulator. **The assignment sheets and provided tables must be returned in the end of the exam!**

Theory 1 (30 Points)

(a) Why is a Gaussian (normal) random distribution of such great importance? Why is noise in digital communication systems distributed according to a Gaussian probability density function (PDF)? Sketch the PDF and the cumulative distribution function (CDF) of a Gaussian random variable.
(b) Why should inter-symbol-interference (ISI) be avoided in digital transmission systems? What criterion has to be fulfilled for ISI-free transmission? How to design the transmit pulse and receiver filter for ISI-free transmission?
(c) What is a bandpass signal? What is the equivalent lowpass representation of a bandpass signal? How can it be interpreted, considering complex-number arithmetic? Define a mathematical relation between the two signal representations and illustrate them in the frequency domain.

Problem 1 (20 Points)

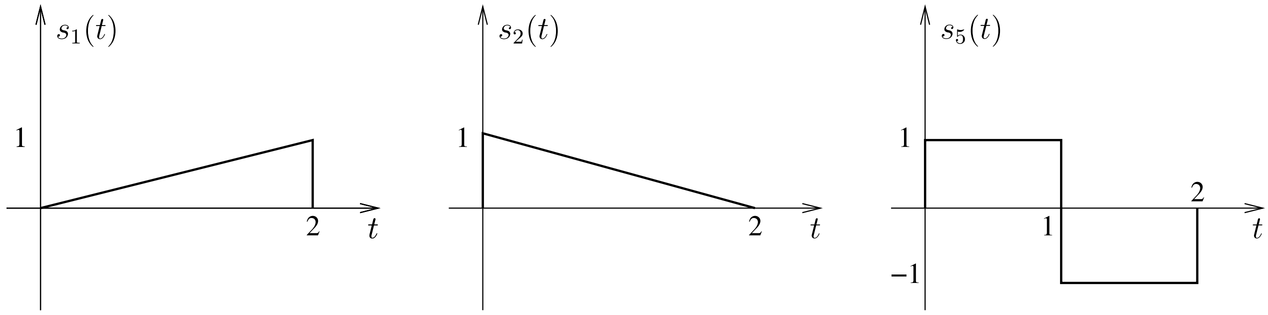
Matched Filter. The pulse shape depicted below is used for data transmission.



(a) Sketch the impulse response of a receiver filter matched to this pulse shape.
(b) Determine the output waveform of this filter for the transmitted pulse shape. (Convolution integral!)
(c) Indicate the sampling time instance that should be used for the signal detection and compute the filter output value at this time instance.

Problem 2 (25 Points)

Given three signals $s_1(t)$, $s_2(t)$ and $s_5(t)$:



(a) Find the norm of $s_1(t)$ and $s_2(t)$ and the inner product of these two signals in a linear space. What is the angle between the two signals?
(b) Find the norm of the signal $s_3(t) = s_1(t) + s_2(t)$ and sketch $s_3(t)$.
(c) Find a signal $s_4(t)$ that is *in* the subspace spanned by $s_1(t)$ and $s_2(t)$ and is orthogonal to $s_3(t)$. Sketch $s_4(t)$.
(d) Find and sketch the signal that lies in the subspace spanned by $s_1(t)$ and $s_2(t)$ and that is closest to $s_5(t)$.
(e) Compute and sketch the projection error depending on t (not the norm of the projection error), resulting from projecting $s_5(t)$ onto the subspace spanned by $s_1(t)$ and $s_2(t)$.

Problem 3 (25 Points)

A random variable X has a Gaussian PDF with mean $m_X = 0$ and variance $\sigma_X^2 = 1$ and random variable Y has the PDF

$$f_Y(y) = 0.3\delta(y - s_1) + 0.7\delta(y - s_2),$$

which can be interpreted as the probability density function of a binary transmission system with symbols $s_1 = -1$ and $s_2 = 1$ (with prior symbol probabilities $P(s_1) = 0.3$ and $P(s_2) = 0.7$.
(a) Illustrate the PDFs of both random variables X and Y.
(b) Compute mean, variance, and second moment for the random variable Y.
(c) Assume a third random variable is defined as $Z = X + Y$. Illustrate the PDF of Z.
(d) Compute mean, variance, and second moment for the random variable Z. Use the moments of the random variables X and Y to get these results.
(e) Compute the conditional PDF $f_{Z|Y}(z|y = s_1)$ for the random variable Z, given that random variable $Y = s_1$.
(f) Given the realization $Z = z$, find a threshold γ for an optimum decision whether $Y = s_1$ or $Y = s_2$, according to the maximum a-posteriori (MAP) criterion.
(g) Determine the error probability (=probability of wrong detection) for the MAP decision.