A Comparison Between Ratio Detection and Threshold Comparison for GNSS Acquisition

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Abstract

In this letter, a comparison between two widespread global navigation satellite system acquisition strategies is presented. The first strategy bases its decision on comparing the energy within a cell to a threshold, while the second one uses the ratio between the two largest cell energies. It is shown that the first method outperforms the second one in terms of receiver operating characteristics in many practically relevant cases. Moreover, despite the purported simplicity of the ratio detection method, it is further shown that its complexity is comparable to or even higher than the one of threshold comparison with adaptive threshold setting.

Index Terms

GNSS, acquisition, receiver operating characteristics, CDMA system, threshold comparison, ratio comparison

I. Introduction

Acquisition is the initial stage in a global navigation satellite system (GNSS) receiver. It is a three-dimensional search, determining the visible satellites, i.e., which spreading codes are received, and providing coarse parameter estimates to the subsequent tracking stage (Doppler frequency, code phase). In its most basic implementation, the detection of a satellite with a particular spreading code is based on the energy contained in a cell. Each cell is a partition of the two-dimensional space specified by a particular Doppler frequency and a particular code phase. Its energy is determined by demodulation and correlation of the received signal with the corresponding carrier and code replicas. The computation of the energies can be performed with some degree of parallelism using standard signal processing tools (e.g., FFTs) [1].

Two strategies for finding the visible satellites are currently used: A threshold comparison method (TC), which compares the energy within each cell to a pre-defined threshold [1], [2], and a ratio detection method (RD), which compares the ratio between the two largest cell energies (of a subset of cells) against a threshold [3]. The rationale for the latter relies upon the conjecture that the ratio between two cell energies is independent of the noise floor, and, consequently, a fixed threshold setting corresponds to a constant false alarm rate [4]. For TC the threshold has to be adapted to the noise floor to obtain a constant false alarm rate.

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While TC is well analyzed in the literature for different search strategies [5]–[10], an evaluation of RD was not possible since the required analytic description has been introduced only recently [11]. Although an analysis is missing, RD is used as a detection method (e.g., [12] or the code supplement to [1]) due to the conjectured simplicity of the algorithm. The main contribution of this letter is thus to provide an extensive comparison of both detection algorithms with respect to performance and complexity.

After introducing the signal model in Section II, Sections III and IV review the detection and false alarm probabilities for TC and RD, respectively. A comparison of receiver operating characteristics of both techniques for various parameter settings in Section VI reveals that – as anticipated in [4] – in many practically relevant cases TC outperforms RD. In addition to that, Section V is devoted to an analysis of the complexity of both approaches.

II. Signal Model

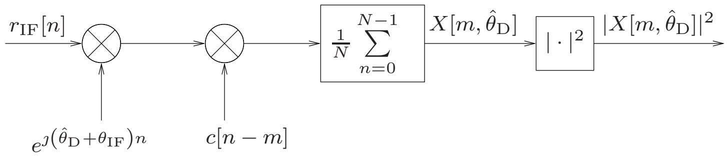

Fig. 1. Acquisition of a signal with unknown Doppler frequency and code phase [11].

The received signal $r_{\mathrm{IF}}[n]$ in the digital intermediate frequency θ_{IF} is demodulated using an estimated Doppler frequency $\hat{\theta}_{\mathrm{D}}$ and correlated with a replica of the expectedly transmitted spreading code $c[n]$, shifted by an expected code phase m. This is illustrated in Fig. 1, where N denotes the number of samples per code period and, therefore, the number of possible code phases. It is assumed that in each Doppler bin at most one cell $X[m,\hat{\theta}_{\mathrm{D}}]$ contains energy from a satellite signal, while all other cells contain noise energy only. This simplification can be easily justified since, first of all, side lobes and cross-correlation levels with different spreading codes can be assumed to be buried below the noise floor (-21 dB according to [13]). Secondly, main lobe widths larger than one sample can be accounted for either by using appropriate decimation methods (e.g., averaging correlation [12], [14]) or by appropriately adapting the search strategy as indicated below.

In [5] it was further assumed that just a single cell of the whole search domain contains signal energy to derive the global detection and false alarm probabilities. This simplification is not necessary for our analysis due to recent results about the influence of Doppler bin widths on GNSS acquisition performance [15].

Since the noise in the received signal is assumed to be zero-mean, white, and Gaussian with two-sided power spectral density $\frac{N_0}{2}$, the decision metric $|X[m,\hat{\theta}_{\mathrm{D}}]|^2$ of noise-only cells is centrally χ^2-distributed with two degrees of freedom. The energy in the signal cell is non-centrally χ^2-distributed with the non-centrality parameter L given by [16]:

$$L = 2T_{\mathrm{per}}\frac{C}{N_0}\mathrm{sinc}^2\left(\frac{\Delta\theta_{\mathrm{D}}}{2\pi}N\right) \tag{1}$$

with T_{per} being the code period and C the carrier power, and where $\Delta\theta_{\mathrm{D}}$ denotes the difference between the actual and the estimated Doppler frequency, i.e., $\Delta\theta_{\mathrm{D}} = \theta_{\mathrm{D}} - \hat{\theta}_{\mathrm{D}}$. The search for a particular spreading code is performed over the two-dimensional region of N possible code phases and K Doppler estimates, i.e., over NK cells in total.

III. Threshold Comparison

From the distribution of $|X[m, \hat{\theta}_{\mathrm{D}}]|^2$ it follows that the probability that the energy of a noise-only cell exceeds a threshold β, i.e., the cell false alarm probability, is [5], [13]

$$P_{fa}(\beta) = \mathrm{e}^{-\frac{2\beta T_{\mathrm{per}}}{N_0}}. \tag{2}$$

Similarly, the probability that the energy of the signal cell exceeds the threshold, i.e., the cell detection probability, can be calculated using [5], [13] as

$$P_{det}(\beta, L) = \mathrm{Q}_1\left(\sqrt{L}, \sqrt{\frac{4\beta T_{\mathrm{per}}}{N_0}}\right) \tag{3}$$

where $\mathrm{Q}_1(\cdot,\cdot)$ is the Marcum Q-function [17].

A thorough analysis of the global detection performance, i.e., the probabilities after a search over N code phases and K Doppler bins, of TC for various search methods (serial, hybrid, and maximum search) can be found in [5]. As it will become clear in Section IV, the RD strategy is most appropriately compared to a hybrid search, which compares the maximum of each subset against a threshold. Multiple cells with signal energy within such a subset do not affect the performance of this strategy, since it bases detection only on the maximum energy cell. Since for the hybrid search no closed form detection probability exists, this letter focuses on the serial search, which performs similarly, but slightly worse than a hybrid search [5]. Assuming that only a single cell contains significant signal energy [5], a serial search over all NK cells yields a global false alarm probability

$$P_{FA}(\beta) = 1 - (1 - P_{fa}(\beta))^{NK} \tag{4}$$

while the global detection probability is

$$P_{DET}(\beta) = \frac{1}{NK}\frac{1 - (1 - P_{fa}(\beta))^{NK}}{P_{fa}(\beta)} P_{det}(\beta, L). \tag{5}$$

Here it is assumed that the signal cell is uniformly distributed over all cells to simplify the derivations. Although the generalization to any other prior distribution for the position of the signal cell can be made with some effort, this analysis was omitted since it neither affects the qualitative statements of this letter, nor does it make the presentation more lucid.

For multiple Doppler bins containing signal cells and a serial search over all code phases for each Doppler bin we have to generalize the expressions according to [15]

$$P_{DET}(\beta) = \frac{P_{det}(\beta, L_0)}{NK}\frac{1 - \overline{P}_{fa}^{N}(\beta)}{P_{fa}(\beta)} \times \left[1 + \sum_{n=1}^{K-1} \overline{P}_{fa}^{n(N-1)}(\beta) \prod_{k=1}^{n} \overline{P}_{det}(\beta, L_k)\right] \tag{6}$$

where $\overline{P}_{fa}(\beta) = 1 - P_{fa}(\beta)$ and $\overline{P}_{det}(\beta, L) = 1 - P_{det}(\beta, L)$. Furhter, L_k denotes the non-centrality parameter of the signal cell in the Doppler bin with index difference k to the correct Doppler bin. In other words, L_0 is the non-centrality parameter for the correct Doppler bin, L_1 for the bins adjacent to the correct, etc.

IV. Ratio Comparison

In RD, the ratio between the largest and the second largest cell energy of a certain subset of cells is compared to a threshold γ. One possibility to partition the NK cells is to form subsets of N cells, one for each of the K Doppler estimates (parallel code phase search, [1, pp. 81]). Recently, for this way of partitioning the false alarm and detection probabilities were obtained analytically. In particular, for subsets with N independent, identically distributed (i.i.d.) noise-only cells the false alarm probability is [11]

$$P_{fa}(\gamma) = (N^2 - N)\mathrm{B}(N-1, 1+\gamma) \tag{7}$$

and the detection probability for subsets containing a signal cell and $N-1$ i.i.d. noise-only cells calculates to [11]

$$P_{det}(\gamma, L) = e^{-\frac{L}{2}} \sum_{q=0}^{\infty} \left[\frac{L^q}{2^q q!} \times {}_{q+2}F_{q+1}\left([-(N-1), \gamma \mathbf{1}_{q+1}]; (\gamma+1)\mathbf{1}_{q+1}; 1 \right) \right] \tag{8}$$

where ${}_pF_q\left([a_1, \ldots, a_p]; [b_1, \ldots, b_q]; c\right)$ is the hypergeometric series, $\mathrm{B}(\cdot, \cdot)$ is the Beta function, and $\mathbf{1}_n$ is a row vector of length n containing ones only. As can be seen from (7), the false alarm probability is independent of $\frac{C}{N_0}$, which indeed allows a fixed threshold setting to obtain a constant false alarm rate. Notice that the subset probabilities for RD are more difficult to evaluate than the cell probabilities for TC. In particular, for TC the cell false alarm probability can be easily solved for β if a desired $P_{fa}(\beta)$ is given, while this does not hold for the beta function of the subset false alarm probability of RD.

If the correlation main lobe is wider than just a single sample, a certain number of samples adjacent to the maximum have to be excluded from the search for the second maximum [1]. In this case, in (8) the number of cells N in the subset has to be reduced accordingly, while (7) turns into a lower bound [11].

Performing a serial search over all K subsets, where only one of these subsets contains a signal cell, yields the global false alarm probability

$$P_{FA}(\gamma) = 1 - (1 - P_{fa}(\gamma))^K \tag{9}$$

and the global detection probability

$$P_{DET}(\gamma) = \frac{1}{K} \frac{1 - (1 - P_{fa}(\gamma))^K}{P_{fa}(\gamma)} P_{det}(\gamma, L). \tag{10}$$

Note that for a maximum search (i.e., a search where the maximum of the whole two-dimensional search region is compared to its second maximum), the global detection and false alarm probabilities are given by (10) and (9), respectively, where N has to be substituted by NK.

In case more than one subset (i.e., more than one Doppler bin if the subsets are chosen accordingly) contains cells with significant signal energy, the global detection probability generalizes to

$$P_{DET}(\gamma) = \frac{1}{K} P_{det}(\gamma, L_0) \left(1 + \sum_{n=1}^{K-1} \prod_{k=1}^{n} \overline{P}_{det}\left(\beta, L_k\right) \right) \tag{11}$$

as it can be shown with the same reasoning as in [5] and [15].

V. Algorithmic Analysis

One argument found in the literature for using the RD algorithm is that the complexity of dividing the maximum by the second maximum is lower than the complexity of calculating the threshold adaptively[1] in the TC algorithm [4]. This Section briefly analyzes the complexity of the two methods. The focus is on a qualitative comparison rather than on a quantitative analysis. Thus, to show that the conjectured superiority of RD over TC in terms of complexity is disputable, we assume an inefficient implementation for TC in which the threshold is adapted in each of the K Doppler bins.

While in Section III the focus was on a serial search, here we will employ a hybrid search for TC. The reason is that RD divides the search space into subsets of cells and uses the subset maximum for detection; for a fair comparison the same is desired for TC.

For the analysis, let us assume that all cell energies of the two-dimensional region are stored in a linear array $x[n]$, $n = 0, \dots, NK-1$. This includes the case where the linear array is only virtual, implemented physically as a circular buffer with N elements which are overwritten for each of the K Doppler bins. Further, we assume that all primitive operations (e.g., addition, comparison of two numbers, writing a variable, indexing an array, etc.) have the same computational complexity. Since both TC and RD perform a serial search over all Doppler bins, the following analysis concentrates on the complexity of detection within one Doppler bin.

Table I sketches the RD algorithm and reveals that for each subset of cardinality N (i.e., for each Doppler bin) between N and $2N$ conditional statements are required to find the first and second maxima within this subset. Computer simulations show that the number of conditional statements is very close to $2N$, since N statements suffice only in the case when the subset is perfectly ordered. Moreover, it turned out that the number of times the body of the conditionals is executed can be neglected. Both conditional statements involve indexing an array and comparing two numbers; in total, two primitive operations each. The ratio comparison itself (line 13) requires a multiplication and a comparison between max_1 and the result of this multiplication, γmax_2. If more than one sample comprises the correlation main lobe, the RD algorithm complicates slightly, since the exclusion of certain samples adjacent to the maximum basically requires a second run over the whole subset.

A possible – ineffective – implementation of a hybrid search with TC and adaptive threshold setting is outlined in Table II. It can be seen that this algorithm requires exactly N conditional statements (again two primitive operations) and N additions for each Doppler bin (indexing, addition, writing; three primitive operations). Furthermore, one subtraction (line 11), one multiplication (line 12), and one comparison (line 13) is needed to determine the presence of a satellite signal. As it is shown in this comparison, this ineffective implementation of TC has a slightly higher complexity due to the cost of the addition. Still, both TC and RD have the same order of complexity, $\mathcal{O}(N)$. Correlation main lobe widths greater than one sample cause only a minor complexity increase of this algorithm, since their effect can be accounted for by subtracting not only the maximum, but also its adjacent samples from the variable sum.

Since in many cases the noise characteristics are changing slowly, it suffices to compute the threshold once for each acquisition process instead of once for every cell subset. This can be done, e.g., by correlating a

[1] An estimate of the noise floor can be obtained by averaging the expected noise-only cell energies, and a threshold can be derived from this by inverting (2) for a desired false alarm probability.

TABLE I

RD ALGORITHM

```
function RD(x[n])
    for k ← 0, K − 1 do
        [max_1, ind_1] ← [0, 0]
        max_2 ← 0
        for n ← 0, N − 1 do
            if x[kK + n] > max_1 then                      ▷ 2 primitive operations
                max_2 ← max_1
                [max_1, ind_1] ← [x[kK + n], kK + n]
            else if x[kK + n] > max_2 then                 ▷ 2 primitive operations
                max_2 ← x[kK + n]
            end if
        end for
        if max_1 > γmax_2 then
            return ind_1                                   ▷ Satellite found, stop
        end if
    end for
    return false                                           ▷ No satellite found, stop
end function
```

spreading code not used for acquisition [2]. In these cases the complexity of TC reduces significantly (the most expensive operation – addition – is performed only once for each acquisition process), while the complexity of RD remains unchanged.

Note that Tables I and II contain another search strategy of [5] as a special case: The maximum search, where the maximum of the two-dimensional search region is either compared to a threshold (TC) or to the second maximum (RD). This corresponds to a partitioning into a single subset ($K \to 1$) which contains all cells of the two-dimensional search region ($N \to NK$). Also in this case the complexity of TC can be assumed lower, if for multiple spreading codes the threshold is computed only once.

Finally, Table III sketches a serial search employing TC, where it is assumed that the threshold has already been computed at the beginning of the acquisition process. As it can be seen, the complexity of such an approach is even lower (but still of order $\mathcal{O}(N)$), since the search stops as soon as the threshold is crossed. A hybrid search would finish the corresponding Doppler bin during the search for its maximum.

VI. PERFORMANCE EVALUATION

The given expressions for the global detection and false alarm probabilities will now be evaluated for a set of system parameters. To make the results comparable, the set of parameters is the same as in [5]. The number of Doppler estimates K is set to 17, which translates to the number of subsets into which the uncertainty region is divided. In each subset, $N = 2046$ cells (code phases) are evaluated, which corresponds to a typical GPS C/A code setting in which each code chip is represented by two samples. Consequently, the code period was set to $T_{\rm per} = 1$ ms and $\frac{C}{N_0}$ was varied between $\{37, 40, 43\}$ dBHz.

To verify the validity of the analytic probabilities, a series of simulations was performed using the specified parameters. To this end, the signal of a single satellite was generated and white Gaussian noise was added

TABLE II

TC ALGORITHM WITH ADAPTIVE THRESHOLD (HYBRID SEARCH)

```
function TC(x[n])
    for k ← 0, K − 1 do
        [max_1, ind_1] ← [0, 0]
        sum ← 0
        for n ← 0, N − 1 do
            sum ← sum + x[kK + n]                        ▷ 3 primitive operations
            if x[kK + n] > max_1 then                    ▷ 2 primitive operations
                [max_1, ind_1] ← [x[kK + n], kK + n]
            end if
        end for
        sum ← sum − max_1                                ▷ Remove signal energy
        β ← c · sum                                      ▷ Compute threshold
        if max_1 > β then
            return ind_1                                 ▷ Satellite found, stop
        end if
    end for
    return false                                         ▷ No satellite found, stop
end function
```

TABLE III

TC ALGORITHM WITH FIXED THRESHOLD (SERIAL SEARCH)

```
function TC(x[n])
    for k ← 0, K − 1 do
        for n ← 0, N − 1 do
            if x[kK + n] > β then                        ▷ 2 primitive operations
                return kK + n                            ▷ Satellite found, stop
            end if
        end for
    end for
    return false                                         ▷ No satellite found, stop
end function
```

according to the considered value of $\frac{C}{N_0}$. The noisy signal was correlated with a local replica of the generated code and with an orthogonal code in order to create specimen of signal and noise-only cells. Consecutively, the obtained two-dimensional uncertainty region was searched serially using TC and RD (the latter subset by subset; see Section IV). In total, the acquisition process was performed $2 \cdot 10^5$ times.

Since each of the 1023 C/A code chips is represented by two samples, the autocorrelation main lobe is three samples wide. This has to be considered in the simulations for a fair comparison: For TC, stopping the serial search at any of the three samples of the correlation main lobe is considered as successful detection, while for RD the immediate neighbors of the maximum sample have to be excluded during the search for the second maximum.

A. Performance for a single Doppler bin containing signal energy

First, it is assumed that only one of the $K = 17$ Doppler bins contains signal energy, i.e., all other Doppler bins contain centrally χ^2-distributed noise-only cells only. A comparison between TC and RD is performed by means of the receiver operating characteristics (ROC), which plot the detection probability as a function of the false alarm probability. Fig. 2 shows the ROCs of both methods for the three simulated values of $\frac{C}{N_0}$. As can be seen, for high global false alarm probabilities (which, according to (2) and (4) correspond to low thresholds) the global detection probability decreases because it is likely that the search is stopped before the correct Doppler bin is reached. Conversely, for high thresholds both detection and false alarm probabilities decrease. Thus, depending on the design criterion (e.g., constant false alarm rate, maximum detection probability, etc.), the current value of $\frac{C}{N_0}$ determines the optimal threshold.

Note that TC outperforms RD over the entire range of false alarm probabilities and for all chosen values of $\frac{C}{N_0}$. In particular, for a global false alarm probability of 0.1 the global detection probability for TC is between 10% and 40% better than for RD, depending on the value of $\frac{C}{N_0}$. As TC was evaluated using a serial search instead of the hybrid search, the difference in performance between TC and RD is even more pronounced than indicated by Fig. 2. Moreover, despite the fact that some of the assumptions stated above do not strictly hold (more than one cell containing signal energy, statistical dependence between adjacent noise-only cells [11]), simulation results show a good match to the analytical descriptions given in Sections III and IV (see Fig. 2). Results for different parameter settings, e.g., a different number K of Doppler bins or a different number N of samples per code phase can be seen in Table IV.

The superiority of TC over RD for the case of a single signal cell is not surprising; if the variance of the noise-only cells would be known, according to the Neyman-Pearson lemma TC would be optimal in the sense of maximizing cell detection probability for a given cell false alarm probability[2]. In our case the variance is not known, but it is estimated to derive a threshold corresponding to a constant false alarm probability (cf. Table II). Thus, TC corresponds to the generalized likelihood ratio test, which for large N is assumed to show only slightly degraded performance compared to the case where the variance is known [18, Chapter 6].

B. Performance for multiple Doppler bins containing signal energy

The case where multiple Doppler bins contain signal cells was also evaluated numerically. To this end, we assumed Doppler bin widths of $W = 500$ Hz, which have been shown to perform well over a wide range of $\frac{C}{N_0}$ in [15]. We further assumed that only the Doppler bins immediately adjacent to the bin corresponding to the correct Doppler frequency contain significant signal energy, i.e., $L_k = 0$ for $k > 1$. Finally, for simplicity it was assumed that the center frequency of the correct bin is identical to the Doppler frequency (i.e., $\Delta\theta_\mathrm{D} = 0$), while for both adjacent bins the Doppler difference is given as

$$\Delta\theta_\mathrm{D} = \frac{2\pi W T_\mathrm{per}}{N} = \frac{\pi}{N}. \tag{12}$$

[2] Thanks to an anonymous reviewer pointing us to this fact.

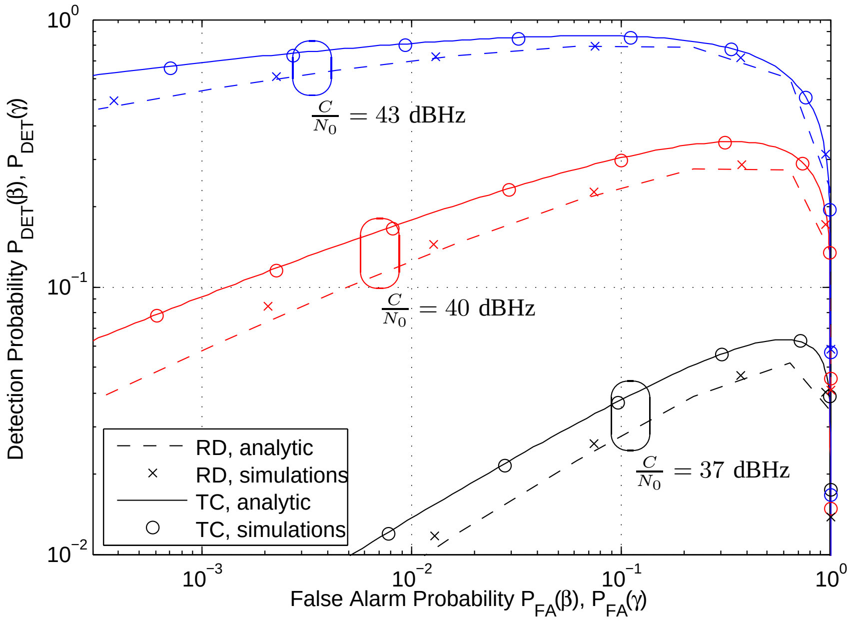

Fig. 2. Comparison of Receiver Operating Characteristics for TC and RD using $N = 2046$ and $K = 17$. Only one Doppler bin contains signal energy.

TABLE IV

PERFORMANCE COMPARISON BETWEEN RD AND TC FOR VARIOUS PARAMETERS ($\frac{C}{N_0} = 40$ dBHz)

Parameters		Detection Probabilities			
Code Phases	Doppler Bins	$P_{FA}(\cdot) = 0.1$		$P_{FA}(\cdot) = 0.01$	
N	K	RD	TC	RD	TC
1023	10	0.31	0.39	0.17	0.245
2046	10	0.27	0.34	0.15	0.205
4092	10	0.235	0.29	0.13	0.17
1023	40	0.215	0.29	0.11	0.17
2046	40	0.185	0.25	0.095	0.14
4092	40	0.16	0.21	0.08	0.115

For the correct bin we thus obtain under the given assumptions $L_0 = 2T_{\text{per}}\frac{C}{N_0}$, while for the adjacent bins we get $L_1 = 2T_{\text{per}}\frac{C}{N_0}\text{sinc}^2(0.5)$. The result of this analysis is shown in Fig. 3, where again it can be seen that TC outperforms RD over all relevant sets of parameters. For high values of $\frac{C}{N_0}$ and high global false alarm probabilities RD achieves better global detection probabilities. This is related to the fact that the detection probability for RD is lower than for TC in each Doppler bin, and thus false detection in bins adjacent to the correct Doppler bin is less likely. However, in all these cases TC achieves a higher maximum for $P_{DET}(\beta)$ at a lower global false alarm probability.

Clearly, this analysis is only valid for a serial search employing TC, since the global detection and false alarm probabilities were derived based upon this assumption (see Section III). Nevertheless, fuelled by the statements in [5] (among all introduced search strategies the serial search performs worst), the authors conclude that in general RD performs worse than TC. A sound comparison of these search strategies, especially in cases where

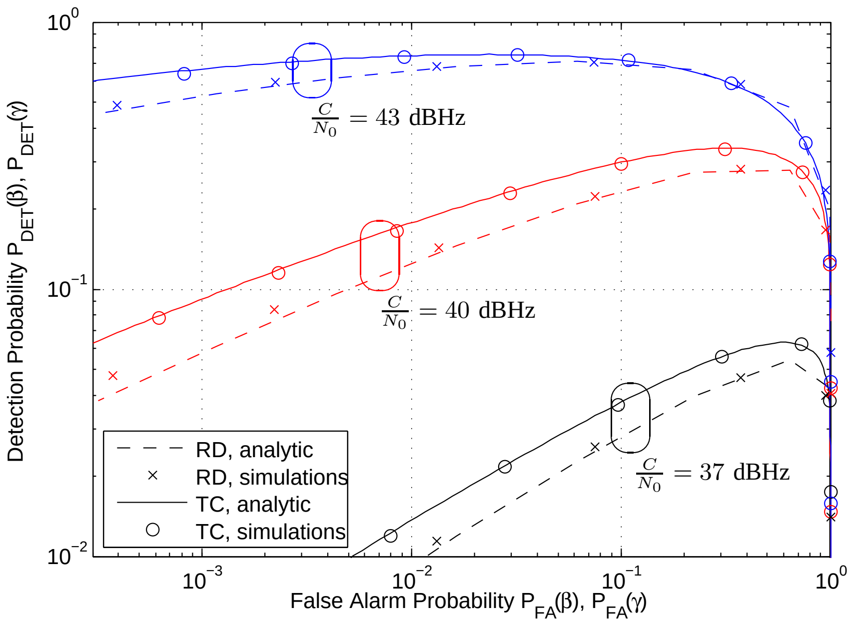

Fig. 3. Comparison of Receiver Operating Characteristics for TC and RD using $N = 2046$ and $K = 17$. In addition to the correct Doppler bin, the two adjacent bins contain significant signal energy.

more than one Doppler bin contains significant signal energy, are within the scope of future work.

VII. Conclusion

This letter compares two widespread signal acquisition strategies: threshold comparison, which compares the energy of every cell against a threshold, and ratio detection, which compares the ratio between the maximum and the second maximum of subsets of all cells against a threshold. Despite the fact that the latter method can use a fixed threshold to obtain a constant false alarm probability, its complexity is comparable to, if not greater than the one of a simple threshold comparison scheme with adaptive threshold computation. Moreover, for many relevant cases it is shown that the receiver operating characteristics (i.e., detection probability depending on the false alarm probability) for a serial search over all cells with threshold comparison are better than for a serial search over subsets of cells employing ratio detection. If and how these statements generalize to other search strategies, e.g., a maximum search, is the object of future work.

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